Math 5440
Monday 11/8

Hw for Friday 11/12:
4.29 1, 2, 4, 7 ← assigned Friday
4.27 1, 3 } new today.
4.28 1, 2 }

Continue Friday notes on nonhomogeneous problems.

projects ?
2^{nd} midterm next Monday. (i.e. a week from today!)

- We may assume the boundary data is homogeneous (= 0), since we already know how to solve the corresponding homogeneous PDEs with non-homogeneous boundary data, and can use superposition.

- Duhamel's principle is really from ODE theory, and says the soln to

$$\begin{cases} u_t = Lu + F & t \geq 0 \qquad (L \text{ linear}) \\ u(0) = 0 \end{cases}$$

is an integral superposition

$$* \quad u(t) = \int_0^t U(\tau)\, d\tau$$

of solns to the homogeneous DE's, $U(t,\tau)$ $t \geq \tau$: solving

$$\begin{cases} u_t = Lu & t \geq \tau \\ u(\tau) = F(\tau) \end{cases}$$

formal check: $u_t = \frac{d}{dt}\int_0^t U(t,\tau)\,d\tau = U(t,t) + \int_0^t U_t(t,\tau)\,d\tau$

$Lu = L\int_0^t U(t,\tau)\,d\tau = \int_0^t L\,U(t,\tau)\,d\tau$ $\quad$ L linear.

so $u_t - Lu = U(t,t) + \int_0^t \underbrace{(U_t - LU)}_{0}\,d\tau = U(t,t) = F(t)$.

$u(0) = 0$ is clear.

If you write the wave eqtn $u_{tt} = c^2\Delta u + F$ as a 1^{st} order system

$(v = u_t)$: $\begin{bmatrix} u \\ v \end{bmatrix}_t = \begin{bmatrix} v \\ \Delta u \end{bmatrix} + \begin{bmatrix} 0 \\ F \end{bmatrix}$ this leads to the wave eqtn soltn in Friday's notes.

the upshot is that for wave eqtn & heat eqtn (in arbitrary space dim's), it suffices to be able to solve the homogeneous WE & HE (with non-homogeneous boundary data)

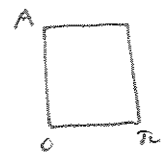
What happens when you try separation of variables (or other techniques) to solve the nonhomogeneous problem

$$\begin{cases} \Delta u = -F & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \\ \quad (\text{or } D_n u = 0) & \end{cases}$$

has a different character however.

Examples

$$\Delta u = -F \qquad 0 \le x \le \pi,\ 0 \le y < A$$
$$u = 0 \qquad \text{on } \partial([0,\pi]\times[0,A])$$



$$u \sim \sum_{n=1}^{\infty} b_n(y) \sin nx$$

$$(\partial_{xx} + \partial_{yy}) u = \sum (b_n'' - n^2 b_n) \sin nx = \sum B_n(y) \sin nx \qquad -F(x,y) \sim \sum_{n=1}^{\infty} B_n(y) \sin nx$$

$$\begin{cases} b_n'' - n^2 b_n = B_n \\ b_n(0) = 0 \\ b_n(A) = 0 \end{cases}$$

- this is a 2nd order linear DE, but it is a boundary value problem and not an initial value problem. There are integral solutions to this sort of BVP, using associated Green's fns

analogous situation on a disk, only with more complicated 2nd order linear boundary value problem

§ 4.27, 4.28.