Review session tomorrow
- you bring questions

Math 5440
Wed Dec 8
Andy will be presenting material related to his work on cloaking theory.

Rough course overview as I see it:

we have modeled (derived) some of the key classical PDE. In almost all cases we linearized to get linear PDE. These PDE have natural initial and boundary value formulations for which we can prove $\exists$, !, regularity theorems.

vector calculus: chain rule, div, grad, curl, change of variables in derivatives and integrals,
FTC, Green's, Stokes' div. thm., flux integrals

include $\mathbb{R}^n$ polar coords
$dV_n = r^{n-1}\,dr\,d\omega_{n-1}$
ω_{S_1}
(this would be nice to clarify.)

PDE derivations
- fluids (Navier-Stokes)
- wave eqtns (1, 2, 3 space variables)
- heat eqtn (n-variables)
- Laplace eqtn
- uniqueness thms via integral identities
    - domains of dependence
- uniqueness via max principle for Laplace & heat eqtns.
    - also estimates using comparison principle
- cov. classification

Solving linear PDE's, BVP's & IBVP's
- scaling cov (e.g. to turn $[0,\ell]$ into $[0,\pi]$)
- separation of variables
- finite superposition (e.g. to isolate PDE, BV's or IV's)
- infinite superposition
    - infinite sums
    - integral solns.
        - when is deriv(sum) = sum(derivs) ?
        - deriv(int) = int(deriv)
    - convolution solns

Fourier series
- def's
- even, odd reflections for cos & sin series
- convergence questions
    - pointwise   f Lipschitz or Hölder cont near x_0
    - uniform   e.g. $f' \in L^2$ implies this quickly.
    - in L^2 $f \in L^2$. "completeness"
    - Fejer & Dini kernels.

Fourier series soltns to PDE's.

1 spaced WE IBVP's. ~ connection to d'Alembert soltn's.
HE

harmonic functions on the disk.
justification that infinite series soltns are soltns
techniques include
justify interchange of derivative and ∞ sum
maximum principle
approximate identity arguments

Green's fcn solutions

1-variable Green's fcns for 2^{nd} order ODE's.
application to $\Delta u = \cancel{f}\, F$ in rectangles & Disk.
Green's function for Δ operator on a domain
(uses fundamental soltns for $\Delta u = f$)
Green's fcn representation for soltn to Special cases

$$\begin{cases} \Delta u = +F & \text{in } \Omega \\ u = h & \text{on } \partial\Omega \end{cases}$$

Hilbert Spaces and the spectral theorem

geometry: inner product, norm, projection to closed subspaces
$T: H \to H$ continuous linear transformation
$\|T\|_{op}$.

ℓ^2
Spectral theorem for compact self-adjoint $T: H \to H$.

application to Δ^{-1} & Green's fcn operators $\leftarrow$ I never got to show compactness for 1 space-d Green's fcn case, it follows quickly from Arzela-Ascoli 5210 (I could sketch)

$L^2(\Omega)$ is isometric to ℓ^2 !
$\uparrow$
connected
open, bd. in $\mathbb{R}^n$

$$f = \sum_{i=1}^{\infty} c_i f_i \longmapsto \{c_i\}_{i \in \mathbb{N}} \in \ell^2.$$

is one generalization of Fourier series.
(another is the eigenfunctions of Laplacian on $S^{n-1} \subset \mathbb{R}^n$, as span of restriction of ~~two~~ degree l homogeneous harmonic polynomials)

solutions of HE & WE using eigenfunctions of Δ, and separation of variables