Math 5440
Friday 8/27

- Derive the formula on p. 3 Wed for the unique soltn to IVP

$$\begin{cases} u_{tt} - c^2 u_{xx} = 0 & x \in \mathbb{R},\ t \geq 0 \\ u(x,0) = f(x) \\ u_t(x,0) = g(x) \end{cases}$$

the discuss

the <u>domain of dependence</u> for computing $u(x_0, t_0)$

the set of x values for which changes in IC's $f(x), g(x)$ can change the value of $u(x_0, t_0)$

the <u>domain of influence</u> for $x_0 \in \mathbb{R}$

the set of points (x,t) with $t \geq 0$ for which $u(x,t)$ can be changed by modifying $g(x_0), f(x_0)$ (by a diff'ble change to f, g.)

the <u>characteristic directions</u> $\langle c, 1\rangle$ and $\langle -c, 1\rangle$, and how they relate to domain of dep & domain of influence.

the operator $\partial_t \partial_t - c^2 \partial_x \partial_x$ "factors" into a composition $(\partial_t - c\partial_x)(\partial_t + c\partial_x)$.

<u>characteristic dirs</u> for an operator are directions for which one of these factors is a multiple of a directional derivative operator.

almost all HW for Friday 9/3
<u>note</u>: partial & complete soltns are in back of text.

1.2 1–5
1.3 1
1.4 1, 2, 8
1.5 1–3

(some will only take a few minutes. others are more challenging)

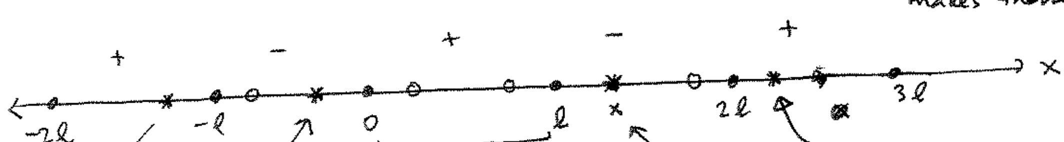
<u>Class exercise 1</u>: prove that a cont. diff'ble fcn $f(x)$ $(x \in \mathbb{R})$ is even iff $f'(x)$ is odd and odd iff $f'(x)$ is even.

$$u(x,t) = \tfrac{1}{2}\big(f(x+ct) + f(x-ct)\big) + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\bar{x})\,d\bar{x}$$

- Solving IBVP

$$\begin{cases} u_{tt} - c^2 u_{xx} = 0 & 0 \le x \le \ell,\ t \ge 0 \\ u(x,0) = f(x) & 0 \le x \le \ell \\ u_t(x,0) = g(x) & \\ u(0,t) = 0 & t \ge 0 \\ u(\ell,t) = 0 & \end{cases}$$

extend f and g so that they are odd extensions at $x=0$ & $x=\ell$. This is the same as saying they're odd wrt every integer multiple of ℓ, and also makes them 2ℓ-periodic



f, g initially defined for $0 \le x \le \ell$

$f(x) := -f(-x)$

$f(x) : -f(\ell - (x-\ell))$
$= -f(2\ell - x)$

$f(x) := -f(2\ell - (x-2\ell))$
$= --f(2\ell - (2\ell - (x-2\ell)))$
$= f(x-2\ell)$

$f(x) := -f(-\ell + (-\ell - x))$
$= -f(-2\ell - x)$
$= f(x+2\ell)$

Then the formula for the soltn shows that $u(x,t)$ is also odd wrt $x=0$ and $x=\ell$ (when considered as a function for $x \in \mathbb{R}$, $t \ge 0$), so satisfies the boundary conditions at $x=0, \ell$

Examples

$$\begin{cases} u_{tt} - c^2 u_{xx} = 0 & 0 \le x \le \pi,\ t \ge 0 \\ u(x,0) = f(x) = \sin x & \\ u_t(x,0) = 0 & 0 \le x \le \ell \\ u(0,t) = 0 & t \ge 0 \\ u(\ell,t) = 0 & \end{cases}$$

$\sin x$ is already odd wrt $x=0$ & $x=\ell$ on all of $\mathbb{R}$.

so

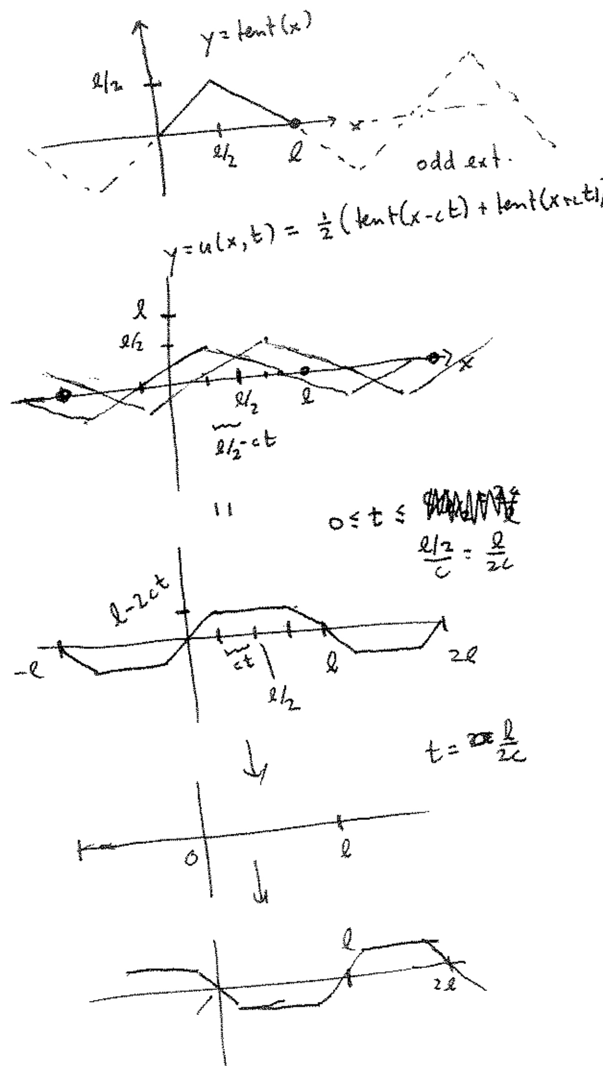
$$u(x,t) = \tfrac{1}{2}\big(\sin(x+ct) + \sin(x-ct)\big) = \sin x \cos ct$$

$\sin x \cos ct + \cancel{\cos x \sin ct}$

$\sin x \cos ct - \cancel{\cos x \sin ct}$

Example

$$\begin{cases} u_{tt} - c^2 u_{xx} = 0 & \\ u(x,0) = f(x) = \text{tent}(x) = \begin{cases} x & 0 \le x \le \ell/2 \\ \ell - x & \ell/2 \le x \le \ell \end{cases} & \\ u_t(x,0) = 0 & 0 \le x \le \ell \\ u(0,t) = 0 & t \ge 0 \\ u(\ell,t) = 0 & \end{cases}$$



see computer.

you can also do "free" bdry conditions by even extension at $x=\ell$, $x=0$ for f, g

$$u_x(0,t) = u_x(\ell,t) = 0 \quad t \ge 0$$

(or have one end fixed and one end free.)