Math 5210
Monday March 9

- Go over Friday's notes, on using convolution to solve the diffusion equation
- Step 3 on page 4 can be justified in the case that the initial temperature distribution has compact support – it's quite a bit like "Theorem 1" for interchanging limits and integrals!

<u>Theorem 1'</u>: Let

$[a,b]\times[c,d] \subset \mathcal{O}_{\text{open}} \subset \mathbb{R}^2$

$f: \mathcal{O} \to X_{\text{Banach}}$ continuous

$\frac{\partial f}{\partial y}(x,y): \mathcal{O} \to X$ continuous

for $y \in [c,d]$

$$g(y) := \int_a^b f(x,y)\,dx$$

<u>Then</u> $g'(y)$ exists, and $g'(y) = \int_a^b \frac{\partial f}{\partial y}(x,y)\,dx$ ("differentiating through the integral sign").

<u>proof</u>

$$\frac{g(y+h)-g(y)}{h} = \frac{1}{h}\left[\int_a^b f(x,y+h)\,dx - \int_a^b f(x,y)\,dx\right]$$

$$= \int_a^b \frac{f(x,y+h)-f(x,y)}{h}\,dx$$

would like to take $\lim_{h\to 0}$ & pass it thru the integral sign – but need to justify this.

e.g. if the difference quotients converge <u>uniformly</u> to $\frac{\partial f}{\partial y}(x,y)$.

well, $\frac{f(x,y+h)-f(x,y)}{h} = \frac{\partial f}{\partial y}(x, c(y))$, $|c(y)-y| \le |h|$, by Mean Value theorem.

And $\frac{\partial f}{\partial y}$ is <u>uniformly</u> continuous on the compact subset $[a,b]\times[c-\delta, d+\delta] \subset \mathcal{O}$ (δ sufficiently small)

So for $\varepsilon > 0$ $\exists\, \delta$ s.t.

$$|c(y)-y| \le |h| < \delta \Rightarrow \left|\frac{\partial f}{\partial y}(x,c(y)) - \frac{\partial f}{\partial y}(x,y)\right| < \varepsilon \quad \forall\, x \in [a,b]$$

then $|h| < \delta$

$$\Rightarrow \left|\frac{g(y+h)-g(y)}{h} - \int_a^b \frac{\partial f}{\partial y}(x,y)\,dx\right| = \left|\int_a^b \frac{\partial f}{\partial y}(x,c(y)) - \frac{\partial f}{\partial y}(x,y)\,dx\right|$$

$$\le \int_a^b |\quad|\,dx < \frac{\varepsilon}{b-a}$$

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