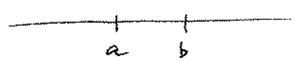
Math 5210 MTWF 2-2:50
Monday 1/12 LCB 215

- syllabus
- textbook available?

HW: review chapter 1, and
be ready to begin with chapter 2 tomorrow!

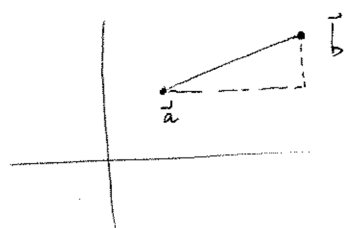
Examples of how to measure distance, in different spaces you may already have experience with

(0) on $\mathbb{R}$



$\text{dist}(a,b) = |b-a|$ absolute value of difference
$d(a,b)$

(1) on $\mathbb{R}^2$, thru sophomore math
you think the only way to measure distance
is the Euclidean one,



$$d_2(\vec{a},\vec{b}) = \|\vec{b}-\vec{a}\|_2 = \left(\sum_{i=1}^{2} (b_i-a_i)^2\right)^{1/2}$$

(X, d) is a metric space if $\exists\ d: X \times X \to \mathbb{R}^+$ s.t.

(1) $d(a,b) \geq 0 \quad \forall a,b;\ d(a,b)=0$ iff $a=b$
(2) $d(a,b) = d(b,a) \quad \forall a,b$ (symmetry)
(3) $d(a,c) \leq d(a,b) + d(b,c) \quad \forall a,b,c.$ (Δ ineq.)

other distances on $\mathbb{R}^2$:

$\text{dist}_1(a,b) = \|a-b\|_1 = \sum_i |a_i-b_i|$ ← Salt Lake City Road distance!

$\text{dist}_\infty(a,b) = \|a-b\|_\infty = \max_i |a_i-b_i|$

$\text{dist}_p(a,b)$

In linear alg & 3210-3220
you verify Δ ineq for d_2, using
Cauchy Schwarz!

is $\|a-b\| \leq \|a-c\| + \|c-b\|$?
$\|x+y\| \leq \|x\| + \|y\|$?

What does the unit disk, $\{P \in \mathbb{R}^2 : d(P,O) = \|P\| < 1\}$, look like
in these different metrics?

In $\mathbb{R}^2$, these "p-metrics" are comparable, in the sense that if
two points are close ~~in~~ wrt one distance, they are close wrt the others.

(2) in ODE's (2280)

$$\begin{cases} \dfrac{d\vec{x}}{dt} = \vec{F}(\vec{x}, t) \\ \vec{x}(0) = \vec{x}_0 \end{cases}$$

space: $\left\{ \vec{y}(t) : [0,1] \to \mathbb{R}^n \;\middle|\; \begin{array}{l} \vec{y}(t) \text{ is continuous} \\ \& \ \vec{y}(0) = \vec{x}_0 \end{array} \right\}$

possible distances:

$$\text{dist}(\vec{y}, \vec{z}) = \max_{t \in [0,1]} \|\vec{y}(t) - \vec{z}(t)\|_2$$ ← best!

or $$\text{dist}(\vec{y}, \vec{z}) = \left(\int_0^1 \|\vec{y}(t) - \vec{z}(t)\|^2 \, dt \right)^{1/2}$$ ← not so good, as it turns out.

(3) in PDE's (2280, 3150)

solve $\begin{cases} \Delta u = 0 & \text{in } D \\ u = \varphi & \text{on bdry of } D \end{cases}$ Dirichlet problem

soltn is a fcn in $\left\{ u : \bar{D} \to \mathbb{R} \;\middle|\; \begin{array}{l} ? \\ u \text{ cont on } \bar{D} \\ u = \varphi \text{ on } \partial D \end{array} \right\}$

$$\text{dist}(u, v) = \max_{x \in D} |u(x) - v(x)|$$

sometimes useful

$$\text{dist}(u, v) = \left(\iiint_D (u(\vec{x}) - v(\vec{x}))^2 \, dV \right)^{1/2}$$

better

$$\text{dist}(u, v) = \left(\iiint_D (u-v)^2 + |\nabla u - \nabla v|^2 \, dV \right)^{1/2}$$

best

(4) Fourier series (2280)

f 2π-periodic

s.t. $\displaystyle\int_{-\pi}^{\pi} f^2(x)\,dx < \infty$.

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos kx \, dx \qquad k = 0, 1, 2, \ldots$$

$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin kx \, dx \qquad k = 1, 2, \ldots$$

$$f \sim \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos kx + \sum_{k=1}^{\infty} b_k \sin kx$$

then for $S_N = \dfrac{a_0}{2} + \displaystyle\sum_{k=1}^{N} a_k \cos kx + b_k \sin kx$,

$\lim_{N \to \infty} \text{dist}(f, S_N) = 0$, where $\text{dist}(f, g) := \left[\displaystyle\int_{-\pi}^{\pi} (f(x) - g(x))^2 \, dx \right]^{1/2}$!