Math 5210-1
Wednesday Feb. 4

HW: for this Friday 2/6, the section 9.3 HW is all postponed to next week.

Finish compactness: Tuesday notes.

Equivalent Metric space characterizations of compactness

H.B. property for A: all open covers $\{\mathcal{O}_\beta\}_{\beta \in B}$ of A have finite subcovers $\bigcup_{j=1}^{N} \mathcal{O}_{\beta_j} \supset A$

$\xLeftrightarrow{\text{Complement version}}$

All collections of closed sets $\{C_\beta\}_{\beta\in B}$ in A with the property that
$$\bigcap_{\beta \in B} C_\beta = \emptyset$$
Have a finite collection with empty intersection:
$$\bigcap_{j=1}^{N} C_{\beta_j} = \emptyset$$

$\Updownarrow$ logical equivalent

Let $\{C_\beta\}_{\beta \in B}$ be closed in A with
$\bigcap_{j=1}^{N} C_{\beta_j} \neq \emptyset$ for every finite subcollection $\{C_{\beta_j}\}_{j=1}^{N}$.

Then $\bigcap_{\beta\in B} C_\beta \neq \emptyset$

this is called the "finite intersection property" for compact sets

$\Updownarrow$ p. 3-5 Tues.

Sequential compactness: $\forall \{a_k\} \subset A\ \exists$ subseq. $\{a_{k_j}\} \to a \in A$

$\Leftarrow\text{-----}\Rightarrow$ complement version of p. 3-5 Tuesday can be done directly

$\Updownarrow$ p 2-3 Tues.

A is complete & totally bd