

Math 5210

Friday April 24

HW due date is Wed April 29

problem session Tuesday after class,
3-4:15 LCB 219.

- Finish Lebesgue integration theory on Wednesday notes.

- Continuous versions of the convergence theorems:

In #18 p. 91 you are asked to prove that if $f^{(t)}: [0,1] \rightarrow \bar{\mathbb{R}}$
 $f^{(t)}(x) := f(x,t)$

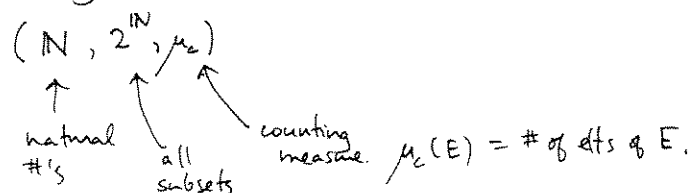
is a family of mslble fns, for each $t \in [0,1]$ (or $[-\varepsilon, \varepsilon]$), $|f^{(t)}(x)| \leq g(x) \forall x \in [0,1]$
and if $g \in L^1([0,1])$.

and if $\lim_{t \rightarrow 0} f^{(t)}(x) = f(x) \quad \forall x \in [0,1]$, then

$$\lim_{t \rightarrow 0} \int_0^1 f(x,t) dx = \int_0^1 f(x) dx$$

Hint Is this not the metric space theorem that a function $H(t)$ is continuous (ε - δ) at t_0 if it's sequentially continuous at t_0 ? Explain.

② An interesting measure space we've seen before.



- $f: \mathbb{N} \rightarrow \mathbb{R}$ is just a sequence! ($f(n) := a_n$)
- What is a simple function?
an integrable simple function?
- What Lebesgue integration theorem implies that a positive (or non-negative) term series may be summed in any order without affecting the result?
- Give an example of a strict inequality for Fatou's Lemma in this space.

- What is a Lebesgue integrable function in this space (on $X = \mathbb{N}$)?
i.e. what kind of series is it?

- What integration theorem immediately implies that if $\sum a_n$ is absolutely convergent, then the series may be summed in any order?

(of course, the theorems above are easier to prove directly from series concepts, but it's interesting & instructive to interpret them as integration theorems.)