Math 5210
Friday April 24

HW due date is <u>Wed</u> April 29
problem session Tuesday after class,
3–4:15 LCB <u>219</u>.

- Finish Lebesgue integration theory on Wednesday notes.

- Continuous versions of the convergence theorems:

<u>In #18 p. 91</u> you are asked to prove that if $f^{(t)}: [0,1] \to \overline{\mathbb{R}}$

$$f^{(t)}(x) := f(x,t)$$

is a family of msble fcns, for each $t \in [0,1]$ (or $[-\varepsilon, \varepsilon]$), $|f^{(t)}(x)| \le g(x)$ $\forall x \in [0,1]$, $g \in L^1([0,1])$.

and if

$$\lim_{t \to 0} f^{(t)}(x) = f(x) \quad \forall x \in [0,1], \text{ then}$$

$$\lim_{t \to 0} \int_0^1 f(x,t)\, dx = \int_0^1 f(x)\, dx$$

<u>Hint</u> Is this not the metric space theorem that a function $H(t)$ is continuous (ε–δ) at t_0 if it's sequentially continuous at t_0? Explain.

(2) An interesting measure space we've seen before.

$$(\mathbb{N},\ 2^{\mathbb{N}},\ \mu_c)$$

natural #'s — all subsets — counting measure. $\mu_c(E) = \#$ of elts of E.

- $f: \mathbb{N} \to \mathbb{R}$ is just a sequence! ($f(n) := a_n$)

- What is a simple function?
  an integrable simple function?

- What Lebesgue integration theorem implies that a positive (or non-negative) term series may be summed in any order without affecting the result?

- Give an example of a strict inequality for Fatou's lemma in this space.

- What is a Lebesgue integrable function in this space (on $X = \mathbb{N}$)?
  i.e. what kind of series is it?

- What integration theorem immediately implies that if $\sum a_n$ is absolutely convergent, then the series may be summed in any order?

(of course, the theorems above are easier to prove directly from series concepts, but it's interesting & instructive to interpret them as integration theorems.)