

MATHEMATICS 5210
Introduction to Real Analysis
Spring 2004

text: *The Way of Analysis*
 by Robert S. Strichartz

when: MTWF 2:00-2:50

where: LCB 121

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office hours: M 11-11:50, T 11-11:50 W 9-10, F 10-10:50

course web site: www.math.utah.edu/~korevaar/5210spring04

course outline:

This is a semester-long course devoted to Real Analysis. As a field of mathematics, Analysis probably arose from the need to either solve, or approximately solve, ordinary differential and partial differential equations in Science and Mathematics. Typically one looks for a solution which is a function of some sort. For example, the trajectory of a rocket can be modeled as the solution to a system of ordinary differential equations which arise from Newton's Laws. In this case the "solution" will be a position function which will depend on a time parameter t . Fluid flows can be modeled as solutions to systems of partial differential equations. Here the "solution" function might consist of instantaneous velocity fields, together with pressure and/or temperature functions, depending on point location and time.

So in trying to model scientific processes one is led to search for solutions which live in various function spaces. Natural questions to ponder are whether the mathematical model you are studying has well-defined solutions, and how to find them or approximate them. And if you approximate them, how good is your approximation? The scientist must also answer the much harder question of how good the model is!

Math 5210 can be thought of as a careful study of the function spaces which most commonly arise in such modeling problems. You may have seen some of these spaces in earlier courses, such as the space of continuous functions on compact intervals, with the sup-norm distance. One of our major focuses, towards the end of the course will be the Lebesgue integral, and the important complete metric spaces known as L^p spaces (chapter 14 in our text, although I will probably also supplement this with material from *Real Analysis* by H. Royden).

In all of these function spaces there are natural ways to measure how close various objects, or "points" are to each other, and these distances satisfy the triangle inequality,

$$\text{dist}(P, Q) \leq \text{dist}(P, R) + \text{dist}(R, Q).$$

Of course, when you were working in Euclidean space, in Math 3210-3220, you were used to writing this in the “norm” notation

$$\|P - Q\| \leq \|R - P\| + \|Q - R\|.$$

A space which has a distance function satisfying a triangle inequality is called a metric space, and analysis related to metric spaces is another main focus of our study this semester. You should think of metric spaces as a useful abstraction which allows one to understand the mathematics which arise in a number of different scientific venues, much as the vector space abstraction which you see in linear algebra allows you to distill mathematics which is common to many disciplines. I plan to begin this course with a review of the real numbers, chapter 2, and then to proceed to metric spaces, chapter 9. As we move through chapter 9, and after we have finished it, we will pursue interesting applications from chapters 7, and 10-13.

Of course this a mathematics offering so sometimes we will forget that what we are doing is related to science and only concentrate on its intrinsic mathematical beauty. But hopefully we will also see how our abstractions can be applied.

prerequisite: Math 3210-3220, Foundations of Analysis. This sequence is a prerequisite not only because of the analysis treatment, but because of the focus on proofs and logic; you will be expected to learn definitions and proofs in Math 5210, and to come up with your own proofs on homework and tests. Most of the material from 3210-3220 is contained in chapters 1-8 of Strichartz, in case you’re rusty. In particular you should immediately make sure you’re comfortable with the preliminary material in Chapter 1.

grading: There will be one midterm and a comprehensive final examination. I plan for the midterm to be taken in class Wednesday March 10, and for the final to be given in our classroom at the University time, Wednesday May 5, 1-3 p.m. There will also be graded homework, which I plan to assign daily and collect on the following Friday. Homework will count 40% of your grade, the midterm will count 20%, and the final will count for 35%. You can earn the remaining 5% of your grade by meeting with me for 15 minutes or so during the first two weeks of the course, so that we can get (re)acquainted.

Unfortunately I am on a University Committee which has course-conflicting meetings scheduled on Tuesdays 1/20, 2/10, 3/2, 3/23, 4/13, so I will probably have to cancel class on those days. I will let you know for sure by the preceding Friday. I have reserved LCB 225 (upstairs) on all Thursdays 2-2:50 for problem sessions, so that you don’t feel cheated. These problem sessions are optional, but many students in previous courses have felt they were very helpful.

It is the Math Department policy, and mine as well, to grant any withdrawal request until the University deadline of Friday March 5

ADA Statement: The American with Disabilities Act requires that reasonable accommodations be provided for students with physical, sensory, cognitive, systemic, learning, and psychiatric disabilities. Please contact me at the beginning of the quarter to discuss any such accommodations for the course.