Math 4530

HW sol'ns 7 - due 4/1

Class exercise I

4.2.3, 4.2.8

I For a graph we know

$$H = \frac{1}{2}\left[\frac{f_{xx}(1+f_y^2) + f_{yy}(1+f_x^2) - 2f_x f_y f_{xy}}{(1+f_x^2+f_y^2)^{3/2}}\right].$$

Show this equals

$$\frac{1}{2}\left[\left(\frac{f_x}{\sqrt{1+f_x^2+f_y^2}}\right)_x + \left(\frac{f_y}{\sqrt{1+f_x^2+f_y^2}}\right)_y\right]$$

Pf

$$\left(\frac{f_x}{\sqrt{\ }}\right)_x = \frac{f_{xx}}{\sqrt{\ }} + f_x(-\tfrac{1}{2})(1+f_x^2+f_y^2)^{-3/2}(2f_x f_{xx} + 2f_y f_{xy})$$

$$= \frac{f_{xx}(1+\cancel{f_x^2}+f_y^2) - \cancel{f_x^2} f_{xx} - f_x f_y f_{xy}}{(\quad)^{3/2}}$$

by symmetry in x & y,

$$\left(\frac{f_y}{\sqrt{\ }}\right)_y = \frac{f_{yy}(1+f_x^2+f_y^2) - f_y^2 f_{yy} - f_y f_x f_{yx}}{(\quad)^{3/2}}$$

adding these two expressions yields

$$\left(\frac{f_x}{\sqrt{\ }}\right)_x + \left(\frac{f_y}{\sqrt{\ }}\right)_y = \frac{f_{xx}(1+f_y^2) - f_x f_y f_{xy} + f_{yy}(1+f_x^2) - f_y f_x f_{yx}}{(1+f_x^2+f_y^2)^{3/2}}$$

$$= \frac{f_{xx}(1+f_y^2) + f_{yy}(1+f_x^2) - 2f_x f_y f_{xy}}{(1+f_x^2+f_y^2)^{3/2}} \quad \blacksquare$$

<u>4.2.3</u> If $f(x,y) = g(x) + h(y)$

then $f_x = g'$ $\quad f_{xx} = g''$

$f_y = h'$ $\quad f_{yy} = h''$

$f_{xy} = 0$

If graph(f) is minimal, set numerator from I = 0, yields

$$g''(1+h'^2) + h''(1+g'^2) = 0$$

$$\frac{g''(x)}{1+g'(x)^2} = \frac{-h''(y)}{1+h'(y)^2}$$

$\Rightarrow$ each expression above is a constant C

$$\frac{g''}{1+g'^2} = C \qquad C \neq 0$$

integrate: $\arctan[g'(x)] = Cx + d$

$$g'(x) = \tan(Cx+d)$$

$$g(x) = \int \frac{\sin(Cx+d)}{\cos(Cx+d)}\,dx = -\frac{1}{C}\ln\cos(Cx+d_1)$$

$C = 0 \Rightarrow g''(x) \equiv h''(y) = 0$

$\Rightarrow g(x) = ax+b$

$h(y) = cy + d$

$f = ax + cy + (b+d)$

<u>plane</u>

We may translate $\left(\tilde{x} = \frac{x-d_1}{C}\right)$

which yields

$$g(x) = -\frac{1}{C} \ln \cos(Cx)$$

Similarly

$$h(y) = \frac{1}{C} \ln \cos(-Cy) = \frac{1}{C} \ln \cos(Cy)$$

$$\Rightarrow f(x) = g(x) + h(y) = \frac{1}{C} \ln \left[\frac{\cos Cy}{\cos Cx}\right]$$

If $C \to -a$

$$f = -\frac{1}{a} \ln \left[\frac{\cos ay}{\cos ax}\right] = \frac{1}{a} \ln \left[\frac{\cos ax}{\cos ay}\right]$$ as written in text.

4.2.8 is a Maple worksheet