

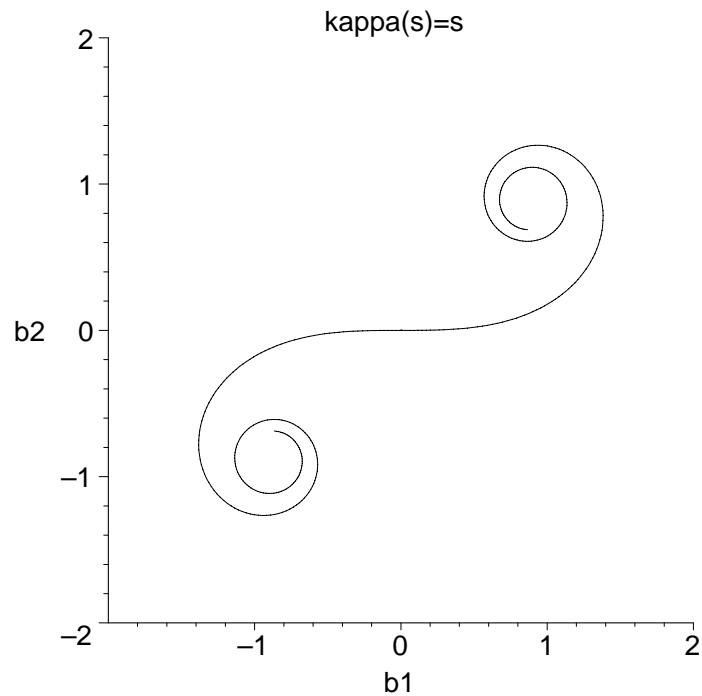
Plane Curves, with prescribed planar curvature

Math 4530-1

Wednesday January 26

For plane curves we can define the normal vector globally as the rotation by $+\pi/2$ of the unit tangent vector, and now there is no restriction that the curvature never be zero. This leads to a simpler Frenet system than for space curves, and you can recreate interesting curves:

```
[ > restart:
[ > with(DEtools):with(plots):
Warning, the name changecoords has been redefined
[ > kap:=t->t: #planar curvature function
a:=-5:b:=5: #s-range
c:=-2:d:=2: #x-range
f:=-2:g:=2: #y-range
[ > sys:=
diff(theta(s),s)=kap(s),
diff(b1(s),s)=cos(theta(s)),
diff(b2(s),s)=sin(theta(s)):
#this is the plane-curve Frenet system
ics:=
theta(0)=0, #start flat
b1(0)=0, #start at origin
b2(0)=0:
p:=dsolve({sys,ics},{theta(s),b1(s),b2(s)},
type=numeric):
p1:=odeplot(p,[b1(s),b2(s)],a..b,numpoints=400,
thickness=1,axes=framed,color=black):
[ > display(p1,view=[c..d,f..g],title='kappa(s)=s');
```



```

> kap:=t->t*sin(t): #planar curvature function
a:=-8:b:=8: #s-range
c:=-2:d:=2: #x-range
f:=0:g:=4: #y-range
> sys:=
    diff(theta(s),s)=kap(s),
    diff(b1(s),s)=cos(theta(s)),
    diff(b2(s),s)=sin(theta(s)):
ics:=
    theta(0)=0, #start flat
    b1(0)=0,    #start at origin
    b2(0)=0:
p:=dsolve({sys,ics},{theta(s),b1(s),b2(s)},
    type=numeric):
p1:=odeplot(p,[b1(s),b2(s)],a..b,numpoints=400,
    thickness=1,axes=framed,color=black):
> display(p1,view=[c..d,f..g],title='kappa(s)=s*sin(s)');

```

