Math 4530
Wed March 2.

If you run out of questions, we can do:

<u>Theorem</u>: The only surfaces for which every point is <u>umbilic</u> ($k_1 = k_2$, i.e. S is a multiple of identity) are pieces of round spheres.

(This shows that you can't arbitrarily specify the shape operator of a surface, so this is not exactly analogous to knowing curvature & torsion of a curve. The analog to $\exists!$ for Frenet systems for curves is called Bonnet's Thm for surfaces with prescribed 1st & 2nd fdl forms - we'll discuss it next week)

<u>pf</u> For a patch we are assuming

$S(X_u) = -U_u = k(u,v)\, X_u$
$S(X_v) = -U_v = k(u,v)\, X_v$

<u>Step 1</u> $k(u,v)$ is const.

<u>pf</u> $-U_{uv} = k_v X_u + k X_{uv}$
$-U_{vu} = k_u X_v + k X_{vu}$

these are $=$, so $k_v X_v = k_u X_u$

X_u, X_v lin ind $\Rightarrow k_v = k_u = 0$
$\Rightarrow k = \text{const}$ (FTC).

<u>Step 2</u> Guess the center of the sphere: (if $k \neq 0$)

$P(u,v) := X(u,v) + \frac{1}{k} U(u,v)$ is const.

<u>pf</u> $\left.\begin{array}{l} P_u = X_u + \frac{1}{k} U_u = X_u + \frac{1}{k}(-k X_u) \equiv 0 \\ P_v = X_v + \frac{1}{k} U_v \equiv 0 \end{array}\right\} \Rightarrow \vec{P} \equiv \text{const}.$

Thus $X(u,v) = P - \frac{1}{k} U(u,v)$
lies on sphere of radius $\left|\frac{1}{k}\right|$ centered at P

If $k \equiv 0$, $U_u \equiv U_v \equiv 0$
$\Rightarrow U$ const
$\Rightarrow X_u \cdot U,\ X_v \cdot U \equiv 0$
$\Rightarrow X(u,v) \cdot U \equiv \text{const}$
$\Rightarrow X$ lies on a plane