Math 4530
Monday 2/28

Exam Friday topics:

Curves
- α pbal
- $\kappa, \tau, \vec{T}, \vec{N}, \vec{B}$
- Frenet system derivation
  - also in the plane
- Derivation of RCE for arbitrary curve

$$\alpha'' = v'(t)\vec{T} + \kappa v^2 \vec{N}$$

and deriving general formulas for κ, τ, N, B. (I could tell you formula for κ or τ & ask you to derive it)

- trickery with the product rule
- basic examples:
  - a-b helix, (circles), circles of curvature
- $\exists!$ theorem for curves of prescribed τ, κ

Surfaces
- (regular) patches $X(u,v)$
- unit normal U
- Shape operator, matrix of shape operator
  - self adjoint
  - principal curvatures and directions (as consequence of spectral thm)
  - H, K
  - normal curvature $k_n(\vec{v})$
  - Interpretation of $[S]$, $k_n(\vec{v})$ in terms of $[D^2 g]$ (Hessian), for a Monge patch above tangent plane
  - I, II, $[\mathrm{I}], [\mathrm{II}]$
  - $[S] = [\mathrm{I}]^{-1}[\mathrm{II}]$ derivation & application.
- Examples
  - graphs
  - surfaces of revolution
  - helicoid

HW comments

- Set 2: $\exists!$ thm for curves ~ see last page of xeroxed sol'ns

- Neat application of Lagrange identity & inverse fcn thm:

Let $P \in M^2$.

M^2 is locally a graph "above" a plane π means:

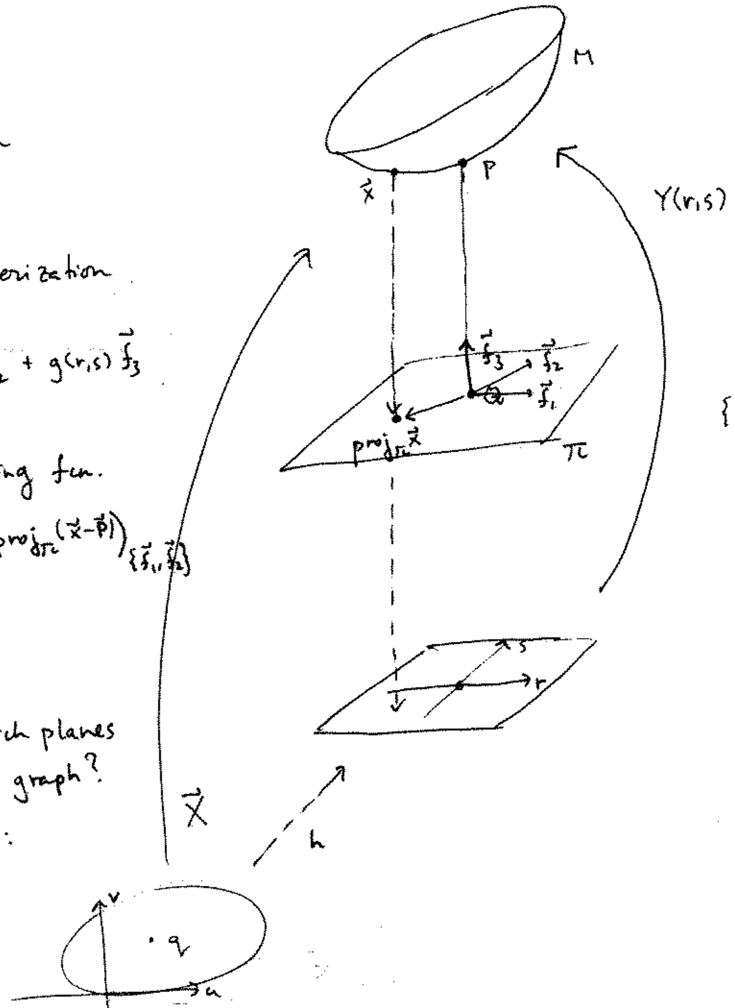
We have a parameterization

$$Y(r,s) = Q + r\vec{f}_1 + s\vec{f}_2 + g(r,s)\vec{f}_3$$

(r,s) near $(0,0)$.

$g(r,s)$ parameterizing fcn.

Note $Y^{-1}(\vec{x}) = \text{coords}\left(\text{proj}_\pi(\vec{x}-\vec{P})\right)_{\{\vec{f}_1,\vec{f}_2\}}$



$\{\vec{f}_1,\vec{f}_2\}$ o.n. basis for π, normal $\vec{f}_3$

$Q := \text{proj}_\pi P$

<u>Question</u>: Above which planes is M^2 locally a graph?

Consider a patch X: $X(q) = P$

define $h(u,v) = \text{coords}\left(\text{proj}_\pi(X(u,v)-P)\right)_{\{\vec{f}_1,\vec{f}_2\}}$

$$= \begin{bmatrix}(X(u,v)-P)\cdot\vec{f}_1 \\ (X(u,v)-P)\cdot\vec{f}_2\end{bmatrix} = \begin{bmatrix} r(u,v) \\ s(u,v)\end{bmatrix}$$

are coords $(Y(r,s)-P) = (r,s)$?

are coords $(X(u,v)-P) = h(u,v)$? ✓

$(u,v) = h^{-1}(r,s)$

$h(u,v) = (r,s)$

$h(q) = (0,0)$

local inverse?

$$h'(u,v) = \begin{bmatrix} r_u & r_v \\ s_u & s_v \end{bmatrix} = \begin{bmatrix} X_u\cdot\vec{f}_1 & X_v\cdot\vec{f}_1 \\ X_u\cdot\vec{f}_2 & X_v\cdot\vec{f}_2\end{bmatrix}$$

$\det = \underbrace{(\vec{X}_u \times \vec{X}_v)}_{\vec{U}} \cdot (\vec{f}_1 \times \vec{f}_2)$ Lagrange!

$\neq 0$ as long as $\vec{U}$ is not in the span of $\{\vec{f}_1, \vec{f}_2\}$

$[h'(u,v)]$ non-sing $\Rightarrow$ local inverse $h^{-1}(r,s)$ exists (π not vertical wrt $\vec{U}$)

<u>Then</u> $Y(r,s) := X(h^{-1}(r,s))$ <u>is</u> a local graphical patch above π!