Math 4530
Monday 4/4

Some minimal surface theorems related to the soap films we'll make today.

<u>Plateau problem</u> : Find a minimal surface spanning a given boundary ~~bo~~ curve (s).

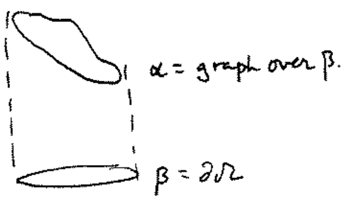
<u>Theorem 1</u> : Let Ω be a <u>convex</u> domain $\mathbb{R}^2$, with boundary $\partial\Omega$ = range $\beta(t)$
Let φ be a (Lipschitz) continuous function on $\partial\Omega$,
Let $\alpha(t) = \langle \beta(t), \varphi(\beta(t)) \rangle$ be its graph.

Then $\exists!$ f s.t.

$$f_{xx}(1+f_y^2) + f_{yy}(1+f_x^2) - 2 f_x f_y f_{xy} = 0$$

i.e. $X(x,y) = \langle x, y, f(x,y) \rangle$ is a minimal monge patch.

<u>Also</u> the graph of f has the <u>least</u> area of all surfaces spanning α.



<u>Theorem 2</u> (Douglas-Rado). Let $\alpha(t)$ describe a simple closed curve in $\mathbb{R}^3$.
Then $\exists$ a (possibly immersed) minimal surface

$$X : \underset{\{u^2+v^2 \le 1\}}{\underset{\shortparallel}{D}} \longrightarrow \mathbb{R}^3 \qquad \text{s.t. } X(\cos\theta, \sin\theta) \text{ reparameterizes } \alpha$$

with the property that X has the minimal area of all maps from D to $\mathbb{R}^3$ which trace out α (possibly reparameterized), as $X(S^1)$.

- There may be more than one such X
- There may be minimal surfaces which are not disks, spanning α.

These theorems are proven variationally - one tries to find a minimum of the surface area functional in the appropriate function space. In <u>analogy</u> to how one can find find harmonic functions u by minimizing $\iint |\nabla f|^2$ in $W^{1,2}$.

On Wed, Fri (at least) we will study another approach to constructing minimal surfaces, involving complex analysis.