

Math 4530
Homework due Wednesday April 10.

1) Exercise 4.4.1

2) Prove that for a subdomain D of a regular surface M , that the integral of 2^*H^*U over the domain (with respect to surface area) equals the integral over the boundary of D of the (exterior) conormal, integrated with respect to arclength. You need only prove this on the image of an isothermal patch, in which case it will follow from the $n=2$ divergence theorem.

3) The inverse to stereographic projection is given by

$$\left[\begin{array}{l} \\ \\ \\ \end{array} \right] \quad X \left(\begin{bmatrix} u \\ v \end{bmatrix} \right) = \begin{bmatrix} 2 \frac{u}{1+u^2+v^2} \\ 2 \frac{v}{1+u^2+v^2} \\ \frac{u^2+v^2-1}{1+u^2+v^2} \end{bmatrix}$$

Show that this gives isothermal coordinates on the sphere (complement of the north pole), and find the length scaling factor ϕ so that

$$\left[\begin{array}{l} \\ \\ \end{array} \right] \quad \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \phi^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Maple is allowed, but not required on this problem.

4) Let $\{\mathbf{f}_1, \mathbf{f}_2\}$ be an orthonormal basis for T_pM . Show that $g:M \rightarrow N$ is conformal (i.e. angle preserving) at P if and only if the map " g star" from T_pM to $T_{g(p)}N$, maps \mathbf{f}_1 and \mathbf{f}_2 to orthogonal vectors with equal, non-zero magnitude. (Hint, recall the argument given for isothermal coordinates on Friday 3/29).

5) Show that $g:M \rightarrow N$ is conformal at P if and only if there is a positive number $\phi^2(P)$ so that

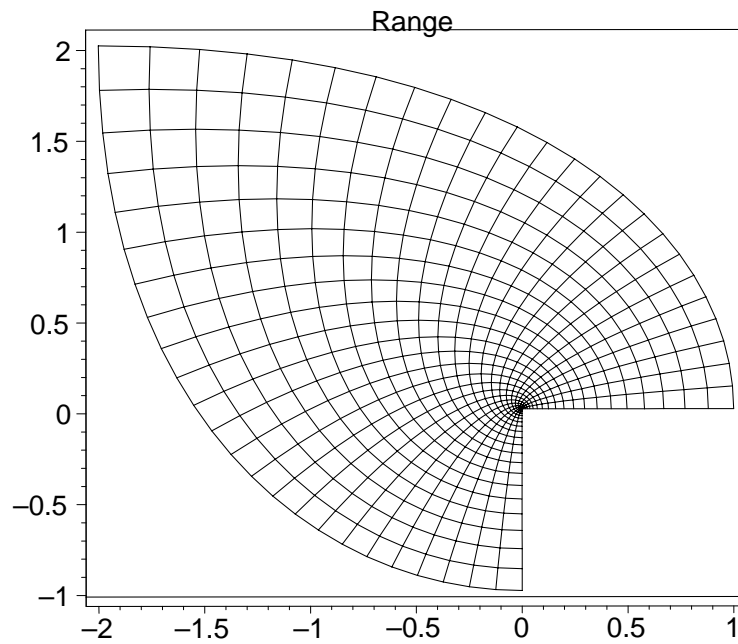
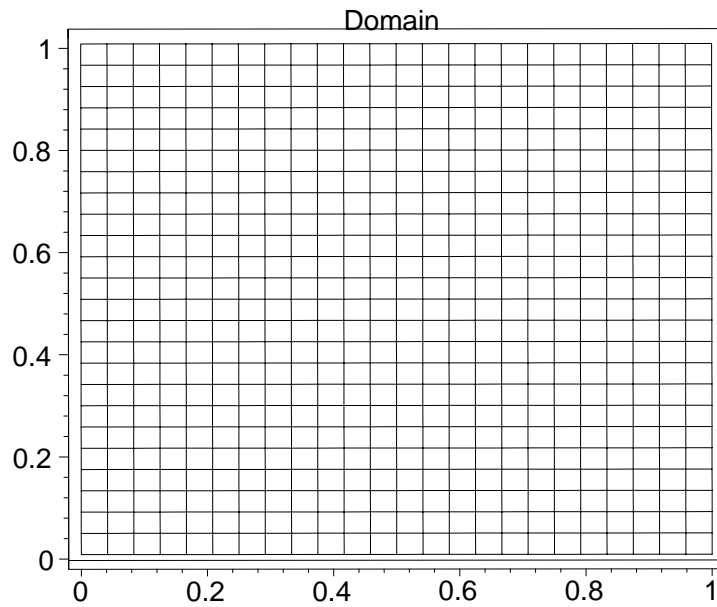
$$\left[\begin{array}{l} \\ \end{array} \right] \quad (g_* v) \cdot (g_* w) = \phi(p)^2 (v \cdot w)$$

for all v, w in T_pM .

6) Exercise 4.3.3: Minimal surfaces have the property that any point has a neighborhood D on which the surface minimizes area among competitors with same boundary (of D). However, the entire minimal surface may not minimize. However, minimal graphs always do minimize surface area. That is what you will show in exercise 4.3.3.

7) For the complex analytic function $g(z)=z^3$, and the point $P=1+i$, write the affine approximation for g at P in complex form and in real form, as is done for $g(z)=z^2$ on page 6 of the Undergraduate Colloquium lecture notes on minimal surfaces. Also draw the "infinitesimal square" picture as is done there. The picture should reflect the scaling and rotation which is done to tangent vectors at P , in order to get their images at $g(P)$. You may use these pictures:

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8) Problems about complex analytic functions: 7.1.2, 7.1.3, 7.1.5, 7.1.6, 7.1.7.

9) Problems using Weierstrass-Enneper: 7.3.5, 7.3.8, 7.3.10, 7.3.18, 7.3.19.

10) Use the formula for Gauss curvature in terms of the Weierstrass data, given in exercise 7.3.15, to compute the Gauss curvature for Enneper's surface (7.3.10), and helcat (7.3.19), by hand. We will prove the Gauss curvature formula in class. Recall how painful this computation was for Maple, at least for some minimal surfaces.