Solving the Frenet System, for prescribed curvature and torsion functions

Math 4530 Spring 02

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[ > restart:
> with(DEtools):with(plots):
    #need DEtools to solve differential equations
Warning, the name changecoords has been redefined
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Here's a pretty self-explanatory procedure which solves the Frenet system, taken more or less from the text.
$>$ recreate3dview: =proc (kap,ta, a,b, c, d, e, f, g,h)
\#kap=curvature, ta=torsion
\#arclength parameter from a to b
\#c..d, e..f, g..h are $x-y-z$ ranges for plot
local
sys, \#the Frenet system
p, \#dummy for ODE solution to Frenet system
ics, \#initial conditions
p1; \#name for ODEplot of $p$
sys:=
diff(alph1 (s), s) =T1 (s),
diff(alph2(s), s)=T2 (s),
diff(alph3(s),s)=T3(s),
diff(T1 (s), s) =kap (s) *N1 (s),
diff(T2 (s), s) $=$ kap (s) *N2 (s),
diff(T3 (s), s) $=$ kap (s) *N3 (s) ,
diff (N1 (s) , s) $=-\mathrm{kap}(\mathrm{s}) * \mathrm{~T} 1(\mathrm{~s})+\mathrm{ta}(\mathrm{s}) * \mathrm{Bl}(\mathrm{s})$,
diff (N2 (s) , s) $=-\mathrm{kap}(\mathrm{s}) * \mathrm{~T} 2(\mathrm{~s})+\mathrm{ta}(\mathrm{s}) * \mathrm{~B} 2(\mathrm{~s})$,
diff (N3 (s) , s) $=-\mathrm{kap}(\mathrm{s}) * \mathrm{~T} 3(\mathrm{~s})+$ ta $(\mathrm{s}) * \mathrm{~B} 3(\mathrm{~s})$,
diff(B1 (s), s) $=-$ ta (s) *N1 (s),
diff (B2 (s), s) $=-$ ta (s) *N2 (s),
diff(B3 (s), s) =-ta(s) *N3 (s);
ics:=
$\operatorname{alph} 1(0)=0, \operatorname{alph} 2(0)=0, \operatorname{alph} 3(0)=0$,
$\mathrm{T} 1(0)=1, \mathrm{~T} 2(0)=0, \mathrm{~T} 3(0)=0$,
$\mathrm{N} 1(0)=0, \mathrm{~N} 2(0)=1, \mathrm{~N} 3(0)=0$,
$\mathrm{B} 1(0)=0, \mathrm{~B} 2(0)=0, \mathrm{~B} 3(0)=1$;
$\mathrm{p}:=\mathrm{dsolve}(\{\mathrm{sys}, \mathrm{ics}\},\{\mathrm{alph}(\mathrm{s}), \operatorname{alph} 2(\mathrm{~s}), \operatorname{alph} 3(\mathrm{~s})$,
T1 (s), T2 (s) , T3 (s) , N1 (s) , N2 (s) , N3 (s) ,
B1 (s), B2 (s), B3 (s) \}, type=numeric) ;
p1:=odeplot (p, [alph1 (s) ,alph2(s), alph3 (s)],a..b,
numpoints=200, thickness=1, axes=boxed, color=black) :
display (p1, scaling=constrained, view=[c..d,e..f,g..h]); end:

Here are some examples:
Example 1: A helix, with constant curvature and torsion
> kap1:=s->.2*s;
tor1:=s->.5;

$$
k a p 1:=s \rightarrow .2 s
$$



