MATHEMATICS 4530-1
Euclidean Curves and Surfaces
SYLLABUS
Spring semester 2002

text: *Differential Geometry and its applications*
by John Oprea
when: MWF 10:45-11:35
where: JTB 120

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office hours: M 2-2:50 p.m., W 2-2:50 p.m., Th 10-11:30.

course home page: www.math.utah.edu/ ~korevaar/4530spring02.html

prerequisites: 2210 or 1260 (Multivariable Calculus), Math 3210-3220 (Introduction to Analysis), or consent of Instructor.

course outline: This course is an introduction to the local and global differential geometry of curves and surfaces. The word "geometry" more or less means "shape", and "differential" means that we will be using differential Calculus tools in describing curve and surface properties. The word "local" means in a neighborhood a a single point, whereas "global" properties depend on the entire curve or surface. We will discuss some beautiful theorems which show how local properties which hold everywhere can have global consequences.

Our plan is to cover most of the first seven chapters of the text, with several additional topics from other source material. Books from which I may plagiarize include *Riemannian Geometry*, by Frank Morgan; *Differential Geometry of Curves and Surfaces* and *Riemannian Geometry*, by Manfredo DoCarmo; *Elementary Differential Geometry*, By Barrett O'Neill.

The shape properties of curves are the topic of Chapter 1; arclength parameterization of regular curves, curvature, torsion, the Frenet frame and the existence and uniqueness theorem for curves. As an interesting diversion we will derive Kepler's Laws of planetary motion. Tools we will review and use in this chapter include the dot and cross products, product rules for vector-valued functions, and the existence and uniquess theorem for systems of differential equations. (You may not have seen this last tool yet unless you've taken a 5000-level course in Analysis or DE's.)

Chapters 2-3 introduce shape properties of surfaces. The natural way to discuss the bending of a surface in 3-space is to study the rates of change of the normal vector to the surface, in various directions. This leads to the "second fundamental form", or "shape operator", which for any point on the surface is a linear operator on the tangent plane there. It turns out that two curvatures: called the mean and Gauss curvature (and equal to half the trace and the determinant of the shape operator), characterize the second order behavior of a surface at a given point on the surface. We will study the geometry of curves on surfaces, learn how to compute various curvatures, and play with intersting examples.

Chapter 4 is an introduction to surfaces of constant mean curvature. These surfaces arise mathematically as critical points for the area functional, with a volume constraint. If there is no volume constraint the mean curvature is zero, and the surfaces are called minimal. Constant mean curvature surfaces arise in nature, for example as thin (soap) films, because such films minimize surface area.

Chapter 5 is an introduction to the branch of mathematics known as intrinsic (or Riemannian) geometry: in this chapter we will study curves on surfaces, their lengths, and curves which are locally length minimizing ("geodesics"). An amazing fact, which we will prove, is that the Gauss curvature mentioned above, actually only depends on this intrinisic geometry, not on how (or if) the surface actually sits in 3-space. In other words, if two surfaces correspond, so that corresponding curves on the surfaces have the same lengths, then the Gauss curvature at corresponding points of the two surfaces is the same. We will discuss hyperbolic and spherical geometry as special cases of intrinsic geometry.

The core of chapter 6 is the Gauss-Bonnet Theorem, an amazing global theorem which we will introduce today: it says that the integral of the Gauss curvature over a surface always gives 2*Pi* the Euler characteristic of the surface - a purely topological quantity.

In chapter 7 we return to the study of minimal surfaces, and explore the relationship between their study and the field of complex analysis. There is a very concrete corrspondence between minimal surfaces and complex analytic functions. (If you've already taken a complex analysis course so much the better, but if you haven't that's O.K. too.)

If time permits we will do a brief unit on relativity, taken from the Frank Morgan book mentioned above. This topic ties in to the ideas of chapter 5.

coursework: Naturally, you will benefit by attending class regularly and by reading the text.

Homework assigned from the book will be collected each week, on Fridays, and a large proportion of the problems will be graded. You will know the assignment for Friday by the Monday of the same week, at the latest. Homework assignments will also be posted on our course web page. I will arrange a *problem session* room for my Thursday "office hours" (10-11:30 a.m.). This will be a time and place for us to meet and work on, or discuss, the homework problems which are due the next day. I encourage you to find classmates with whom you can also discuss homework and other topics. Of course I am also available during my regular office hours, or if necessary by appointment.

There will be two in-class midterms (closed book), as well as a final exam with the same constraint. The dates are as follows:

exam 1: Friday March 8. Probable course material from chapters 1-3.
exam 2: Friday April 6. Probable course material from chapters 4-6.
Final Exam: Wednesday May 8, 10:30 a.m. - 12:30 p.m. in our classroom. The exam will cover the entire course. This is the University-scheduled time.

Instead of taking the final exam you may choose to complete a project related to the course material. This project will have a written component, and I may also encourage you to make a class presentation if time permits. I will be happy to discuss topic ideas with you, there are many directions you can go in depending on your interests. For example, several students last year opted for the project route, with titles taken from their particular interests: Hermite Bicubic Interpolation, Geometry of General Relativity, Celtic Knots.

grading: Each midterm will count for 20% of your grade, the homework will count for a total of 30% of your grade, and the final exam or project will make up the remaining 25% of your grade. You earn the remaining 5% of your grade by meeting with me for 20 minutes sometime within the first two weeks of class, so that we can introduce (or reintroduce) ourselves.

It is the Math Department policy, and mine as well, to grant any withdrawl request until the University deadline of Friday March 2.

ADA statement: The American with Disabilities Act requires that reasonable accomodations be provided for students with physical, sensory, cognitive, systemic, learning, and psychiatric disabilities. Please contact me at the beginning of the semester to discuss any such accommodations for the course.