Math 4530

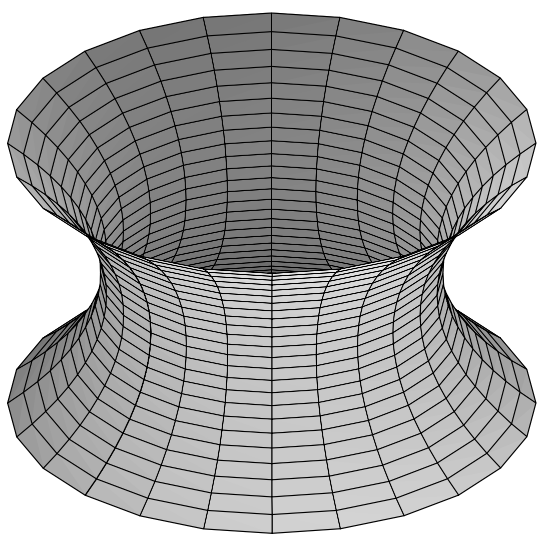
homework due March 30 - parts of Oprea 4.2.5, 4.3.4

4.2.5 -graphing part; computations for Gauss and mean curvature are due this Friday, April 5

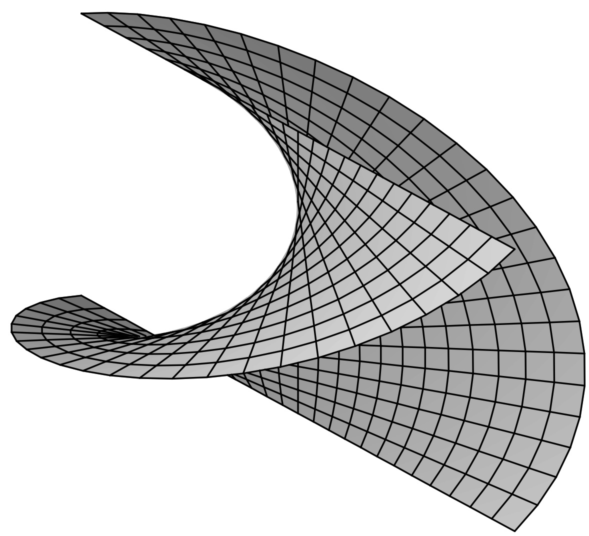
```
>
> restart:
> with(plots):
Warning, the name changecoords has been redefined

> x:=(u,v,t)->[cos(t)*sinh(v)*sin(u)+sin(t)*cosh(v)*cos(u),
     -cos(t)*sinh(v)*cos(u)+sin(t)*cosh(v)*sin(u),
     u*cos(t)+v*sin(t)];
  #the book calls this family helcat because it is a 1-parameter
  #family of surfaces which interpolates the catenoid to the
  helicoid.
  #amazingly, the gij matrices for each of these surfaces are the
  same!
```

$$x := (u, v, t) \to [\cos(t)\sinh(v)\sin(u) + \sin(t)\cosh(v)\cos(u),\ -\cos(t)\sinh(v)\cos(u) + \sin(t)\cosh(v)\sin(u),\ u\cos(t) + v\sin(t)]$$

```
> plot3d(x(u,v,Pi/2),u=0..2*Pi,v=-1..1);
  #catenoid
```



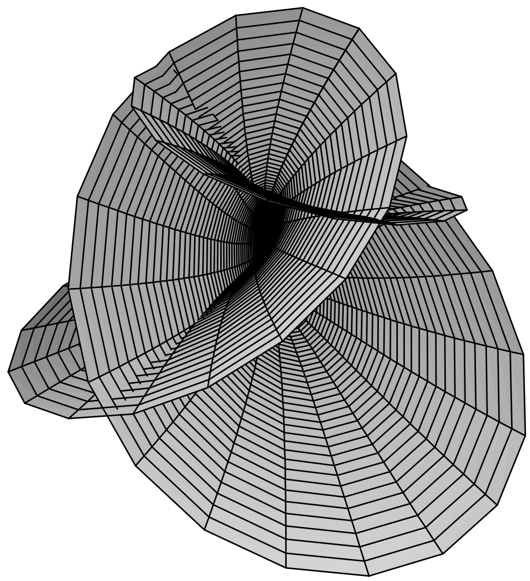
```
> animate3d(x(u,v,t),u=0..Pi,v=-1..1,t=0..Pi/2,frames=30);
  #you can make a movie of the transformation, that's what this
  #command does.  If you try it from Maple you can mouse on the
  #output and menu items appear which let you look at the animation
```



```
> hen:=(u,v)->[2*sinh(u)*cos(v)-2/3*sinh(3*u)*cos(3*v),
      2*sinh(u)*sin(v)-2/3*sinh(3*u)*sin(3*v),
      2*cosh(2*u)*cos(2*v)];
  #Henneberg's surface
```

$$hen := (u, v) \to \left[2 \sinh(u) \cos(v) - \frac{2}{3} \sinh(3\,u) \cos(3\,v),\ 2 \sinh(u) \sin(v) - \frac{2}{3} \sinh(3\,u) \sin(3\,v),\ 2 \cosh(2\,u) \cos(2\,v)\right]$$

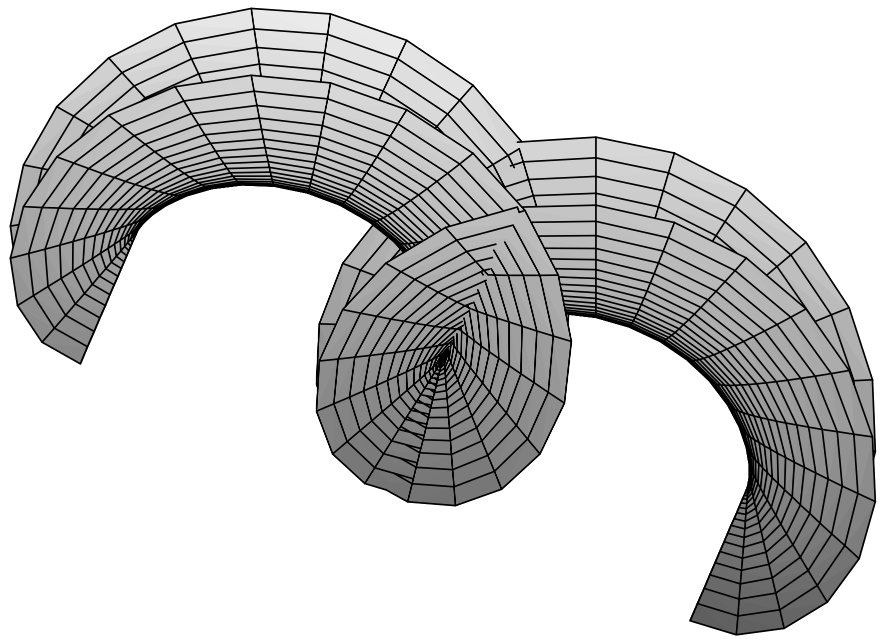
```
> plot3d(hen(u,v),u=0..1,v=0..2*Pi,grid=[50,50]);
```



```
> cata:=(u,v)->[u-sin(u)*cosh(v),1-cos(u)*cosh(v),4*sin(u/2)*sinh(v/
  2)];
  #Catalan’s surface
```

$$cata := (u, v) \to \left[u - \sin(u)\cosh(v),\ 1 - \cos(u)\cosh(v),\ 4\sin\left(\frac{1}{2}u\right)\sinh\left(\frac{1}{2}v\right) \right]$$

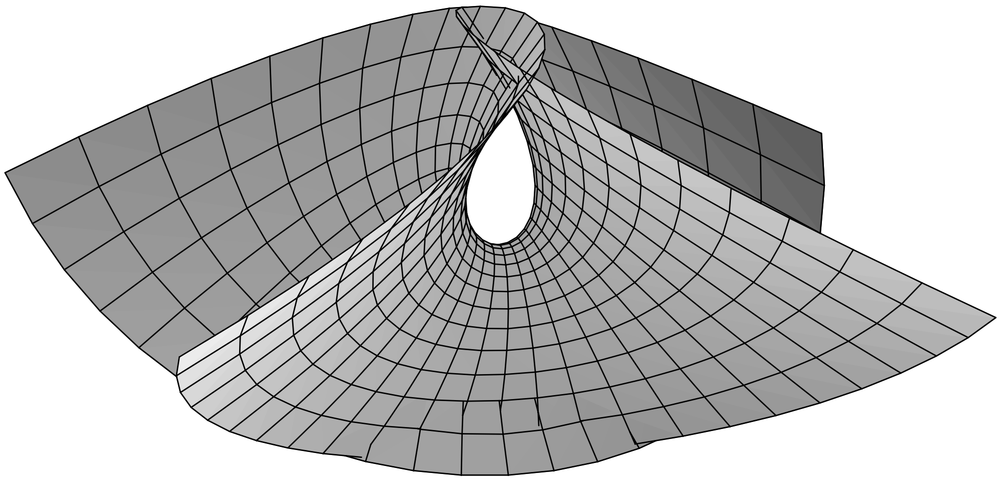
```
> plot3d(cata(u,v),u=0..4*Pi,v=-2..2,grid=[40,40]);
```



```
> enn:=(u,v)->[u-u^3/3+u*v^2,v-v^3/3+v*u^2,u^2-v^2];
  #Enneper's surface
```

$$enn := (u, v) \rightarrow \left[u - \frac{1}{3}u^3 + u\,v^2,\, v - \frac{1}{3}v^3 + v\,u^2,\, u^2 - v^2\right]$$

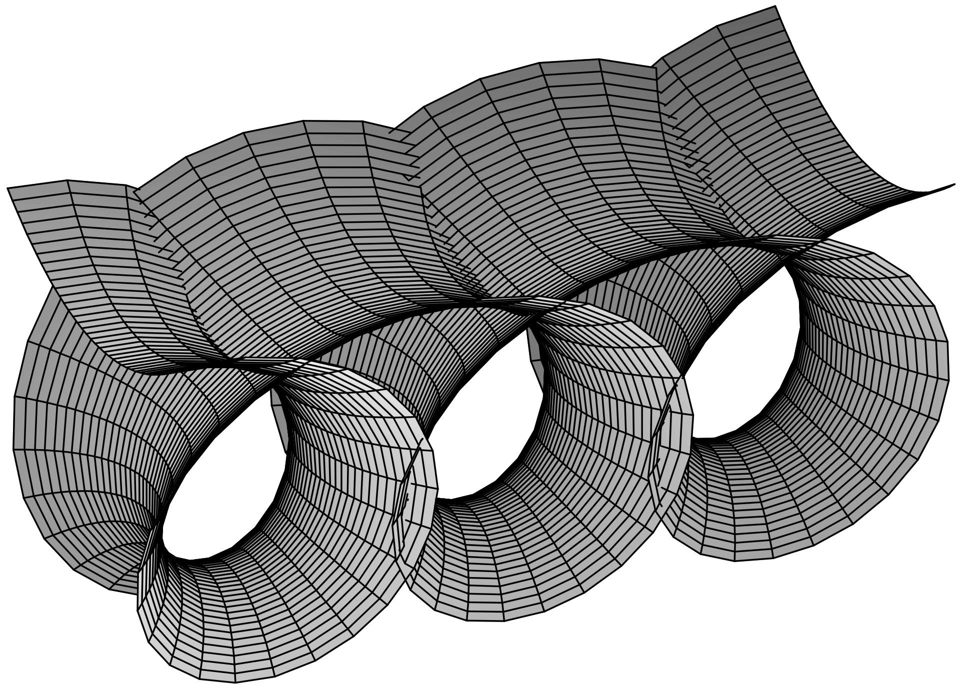
```
> plot3d(enn(u,v),u=-2..2,v=-2..2);
```



```
> plan:=(u,v,c)->[(c*u+sin(u)*cosh(v))/sqrt(1-c^2),
  (v+c*cos(u)*sinh(v))/sqrt(1-c^2),
  cos(u)*cosh(v)];
  #planar lines of curvature
```

$$plan := (u, v, c) \to \left[\frac{c\,u + \sin(u)\cosh(v)}{\sqrt{1-c^2}}, \frac{v + c\cos(u)\sinh(v)}{\sqrt{1-c^2}}, \cos(u)\cosh(v) \right]$$

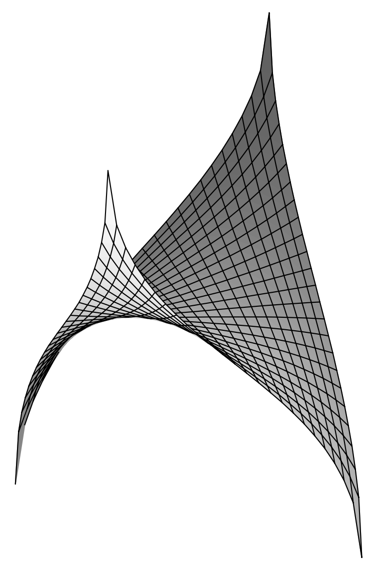
```
> plot3d(plan(u,v,.3),u=0..6*Pi,v=-1..1,grid=[80,40]);
```



```
> sherk5:=(u,v)->[arcsinh(u),arcsinh(v),arcsin(u*v)];
  #Sherk’s surface
```

$$sherk5 := (u, v) \rightarrow [\operatorname{arcsinh}(u), \operatorname{arcsinh}(v), \arcsin(v\,u)]$$

```
> plot3d(sherk5(u,v),u=-1..1,v=-1..1);
```



4.3.4

```
> f:=(x,a)->a*cosh(x/a);
```

$$f := (x, a) \to a \cosh\left(\frac{x}{a}\right)$$

```
> A:=solve(f(.6,a)=1,a);
```

$$A := .7450710899$$

```
> SA:=2*2*evalf(Pi)*int(sqrt(1+diff(f(x,A),x)^2),x=0..(.6));
  #surface area formula for a rotated graph, from multivariable
  Calculus
```

$$SA := 8.381581464$$

```
> 2*evalf(Pi);
  #Area of two radius 1 disks, notice this is less than SA above
```

$$6.283185308$$