Delaunay Surface profile curves
Math 4530
March 28, 2001

We solve the differential equations for a surface of revolution having mean curvature = 1/2. Solutions for the profile curve which start out being parameterized by arclength continue to be so.

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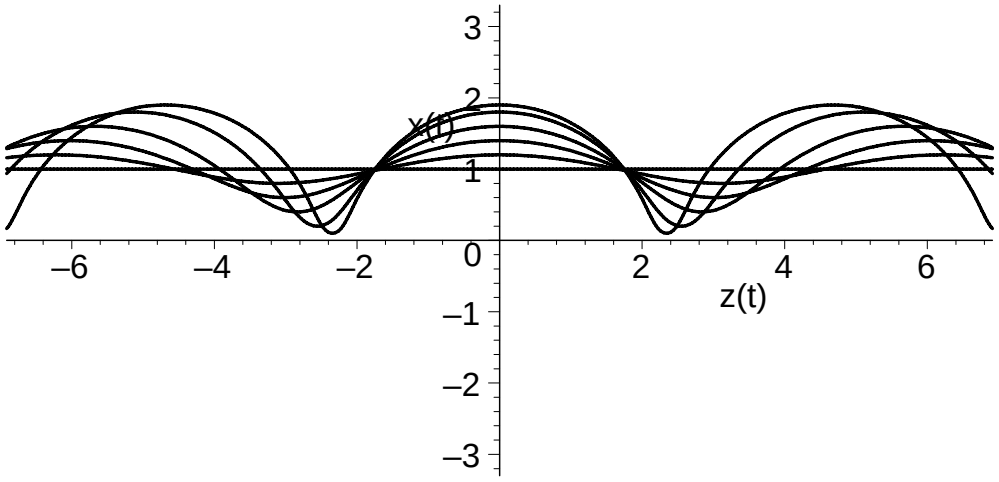
> 
> 
> with(DEtools):

Warning, the name adjoint has been redefined

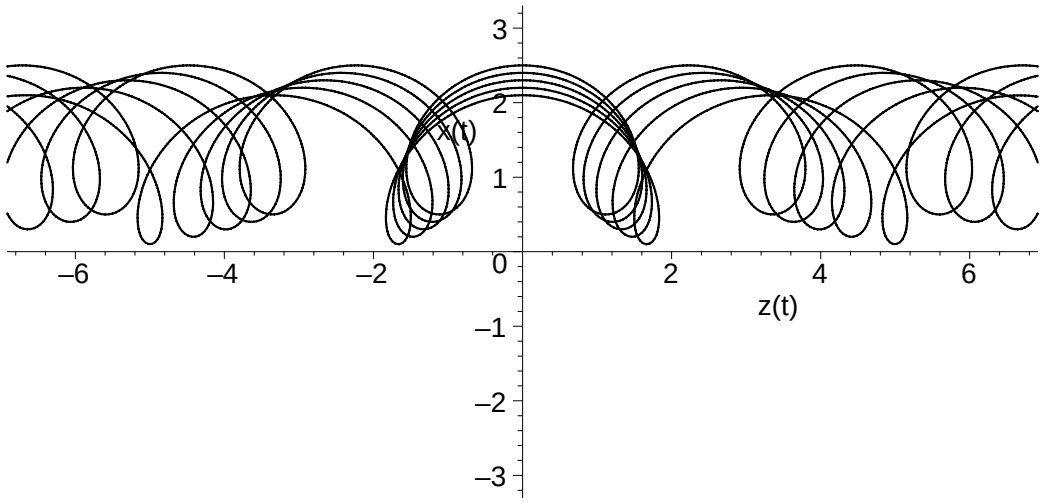
> deqtn:={diff(x(t),t,t)-diff(z(t),t)^2/x(t) = -diff(z(t),t),
          diff(z(t),t,t)+diff(z(t),t)*diff(x(t),t)/x(t) =
  diff(x(t),t)};
```

$$deqtn := \left\{ \left(\frac{\partial^2}{\partial t^2} \mathrm{z}(t) \right) + \frac{\left(\frac{\partial}{\partial t} \mathrm{z}(t) \right) \left(\frac{\partial}{\partial t} \mathrm{x}(t) \right)}{\mathrm{x}(t)} = \frac{\partial}{\partial t} \mathrm{x}(t), \left(\frac{\partial^2}{\partial t^2} \mathrm{x}(t) \right) - \frac{\left(\frac{\partial}{\partial t} \mathrm{z}(t) \right)^2}{\mathrm{x}(t)} = -\left(\frac{\partial}{\partial t} \mathrm{z}(t) \right) \right\}$$

```
> ICS1:=[[x(0)=1,z(0)=0,D(x)(0)=0,D(z)(0)=1],
         [x(0)=1.2,z(0)=0,D(x)(0)=0,D(z)(0)=1],
         [x(0)=1.4,z(0)=0,D(x)(0)=0,D(z)(0)=1],
         [x(0)=1.6,z(0)=0,D(x)(0)=0,D(z)(0)=1],
         [x(0)=1.8,z(0)=0,D(x)(0)=0,D(z)(0)=1],
         [x(0)=1.9,z(0)=0,D(x)(0)=0,D(z)(0)=1]]:
> DEplot(deqtn,[x(t),z(t)],t=-20..20,ICS1,x=-3..3,z=-2*Pi..2*Pi,
  linecolor=black,scene=[z(t),x(t)],stepsize=.05);
  #embedded Delaunay surfaces
```




```
> ICS2:=[[x(0)=2.1,z(0)=0,D(x)(0)=0,D(z)(0)=1],
         [x(0)=2.2,z(0)=0,D(x)(0)=0,D(z)(0)=1],
         [x(0)=2.3,z(0)=0,D(x)(0)=0,D(z)(0)=1],
         [x(0)=2.4,z(0)=0,D(x)(0)=0,D(z)(0)=1],
         [x(0)=2.5,z(0)=0,D(x)(0)=0,D(z)(0)=1]]:
> DEplot(deqtn,[x(t),z(t)],t=-20..20,ICS2,x=-3..3,z=-2*Pi..2*Pi,
  linecolor=black,scene=[z(t),x(t)],stepsize=.05);
  #immersed Delaunay surface profiles
```


3
2
1
0
–1
–2
–3
x(t)
–6
–4
–2
2
4
6
z(t)