

**MATHEMATICS 4530-1**  
**Euclidean Curves and Surfaces**  
**SYLLABUS**  
Spring semester 2001

**text:**        *Differential Geometry of Curves and Surfaces*  
                  by Manfredo P. Do Carmo

**when:**       MWF 10:45-11:35

**where:**      NS 201

**instructor:**       Prof. Nick Korevaar

**office:**         JWB 218

**telephone:**     581-7318

**email:**         korevaar@math.utah.edu

**office hours:**   M 2-2:50 p.m., T 10:00-11:50 a.m., W 2-2:50 p.m., Th TBA.

**course home page:**   www.math.utah.edu/~korevaar/4530spring01.html

**prerequisites:** 2210 or 1260 (Multivariable Calculus), Math 3210-3220 (Introduction to Analysis), or consent of Instructor.

**course outline:** This course is an introduction to the local and global differential geometry of curves and surfaces. The word "geometry" more or less means "shape", and "differential" means that we will be using differential Calculus tools in describing curve and surface properties. The word "local" means in a neighborhood a a single point, whereas "global" properties depend on the entire curve or surface. We will discuss some beautiful theorems which show how local properties which hold everywhere can have global consequences.

Our plan is to cover the first four chapters of the text, with several additional topics from chapter 5 and elsewhere.

The shape properties of curves are the topic of Chapter 1; arclength parameterization of regular curves, curvature, torsion, the Frenet frame and the existence and uniqueness theorem for curves. Additional topics may include Kepler's Laws of planetary motion, Bezier splines. Tools we will review and use in this chapter include the dot and cross products, product rules for vector-valued functions, and the existence and uniqueness theorem for systems of differential equations. (You may not have seen this last tool yet unless you've taken a 5000-level course in Analysis or DE's.)

Chapter 2 is about first order properties of surfaces, starting with the definition of a regular surface, coordinates on surface, change of coordinates, differentiable functions on a surface, differentials of maps, orientation, and the first fundamental form and area form for a surface. Important tools we will review and use include the chain rule and the inverse/implicit function theorem from Math 3220.

Chapter 3 is about the second order behavior which describes locally how embedded surfaces "curve" in  $\mathbb{R}^3$ , so-called "extrinsic geometry." We will study the Gauss map, the shape operator (second fundamental form), and the mean curvature and Gauss curvature of surfaces. This will include some further discussion of minimal surfaces, ruled surfaces, and surfaces of revolution.

Chapter 4 is an introduction to the "intrinsic geometry" of surfaces, that is geometric properties which can be discerned just by measuring distances within the surface, and not necessarily having to do with how the surface is embedded into 3-space. We will study surface isometries, geodesics, Gauss' Theorem, parallel transport, and the Gauss-Bonnet Theorem and some applications.

The material in this course leads in a number of different directions. The chapter 2 material is a concrete introduction to the study of differentiable manifolds, which arise in many different

contexts. Minimal surfaces and related objects from Chapter 3 arise in science from the Calculus of Variations: they are optimal in some sense, and so they appear in nature. There are also surprising connections to Complex Analysis. Chapter 4 can be thought of as an introduction to the field of Riemannian Geometry. Riemannian geometry and geodesic curves are part of the study or Relativity in Physics.

**coursework:** Naturally, you will benefit by attending class regularly and by reading the text.

*Homework* assigned from the book will be collected each week, on Fridays, and a large proportion of the problems will be graded. You will know which problems will be collected on Friday by the Monday of the same week, at the latest. We will arrange a *problem session* time on Thursdays, for those of you who would like to have a place to discuss the homework together and/or with our course assistant. Of course I am also available during office hours, or if necessary by appointment.

There will be two in-class midterms (closed book), as well as a final exam with the same constraint. The dates are as follows:

**exam 1:** Wednesday February 21. Probable course material is chapters 1-2.

**exam 2:** Friday April 6. Probable course material is chapters 3-4.

**Final Exam:** Tuesday May 1, 9:15-11:15 a.m. in our classroom. The exam will cover the entire course. This is the University-scheduled time.

Instead of taking the final exam you may choose to complete a project related to the course material. This project will have a written component, and I may also encourage you to make a class presentation. I will be happy to discuss topic ideas with you; there are many directions you can go in depending on your interests.

**grading:** Each midterm will count for 20% of your grade, the homework will count for a total of 30% of your grade, and the final exam or project will make up the remaining 25% of your grade. You will earn the remaining 5% of your grade by meeting with me for 20 minutes sometime in the first two weeks of class, so we can introduce ourselves.

It is the Math Department policy, and mine as well, to grant any withdrawal request until the University deadline of Friday March 2.

**ADA statement:** The American with Disabilities Act requires that reasonable accommodations be provided for students with physical, sensory, cognitive, systemic, learning, and psychiatric disabilities. Please contact me at the beginning of the semester to discuss any such accommodations for the course.