Math 4530
Playing with plane curves
Friday Januray 26

We use the formulas from #9 on page 24 to plot planar curves with prescribed planar curvature:

```
> restart:with(plots):
Warning, the name changecoords has been redefined
```

```
> theta:=t->int(k(r),r=0..t)+phi;
     #the integral version of k=
     #d/ds of theta
```

$$\theta := t \to \int_0^t \mathrm{k}(r)\,dr + \phi$$

```
> F:=s->int(cos(theta(t)),t=0..s)+a;
  G:=s->int(sin(theta(t)),t=0..s)+b;
     #here alpha(s)=[F(s),G(s)]
```

$$F := s \to \int_0^s \cos(\theta(t))\,dt + a$$

$$G := s \to \int_0^s \sin(\theta(t))\,dt + b$$

```
> a:=0;b:=0;
    #start at origin
  phi:=0;
    #start with theta(0)=0
```

$$a := 0$$

$$b := 0$$

$$\phi := 0$$

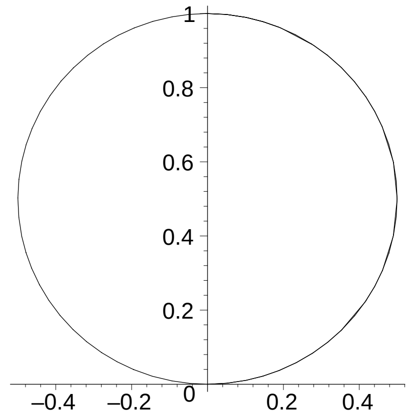
```
> k:=r->2;
     #constant curvature 2
```

$$k := 2$$

```
> [F(s),G(s)];
```

$$\left[\frac{1}{2}\sin(2\,s), -\frac{1}{2}\cos(2\,s)+\frac{1}{2}\right]$$

```
> plot([F(s),G(s),s=0..3*Pi/2],color=black);
    #should be circle of radius 1/2.
```



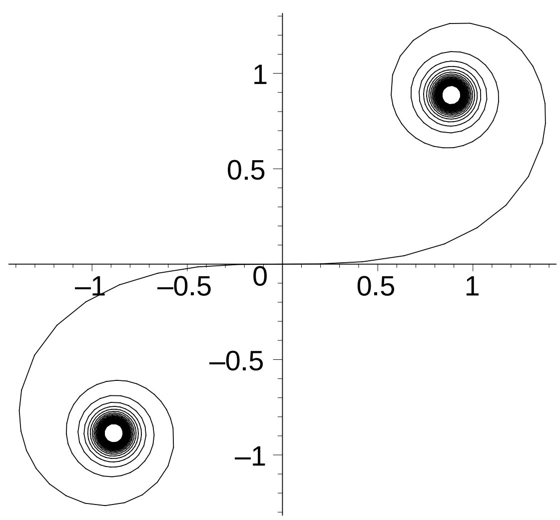

```
> k:=r->r;
   #should be spiral-like
```

$$k := r \to r$$

```
> [F(s),G(s)];
   #amazingly you get a symbolic form for the path
```

$$\left[\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{s}{\sqrt{\pi}}\right), \sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{s}{\sqrt{\pi}}\right)\right]$$

```
> plot([F(s),G(s),s=-20..20],color=black);
```




```
>
> k:=r->exp(r);
  theta(t);
  F(s);
  G(s);
```

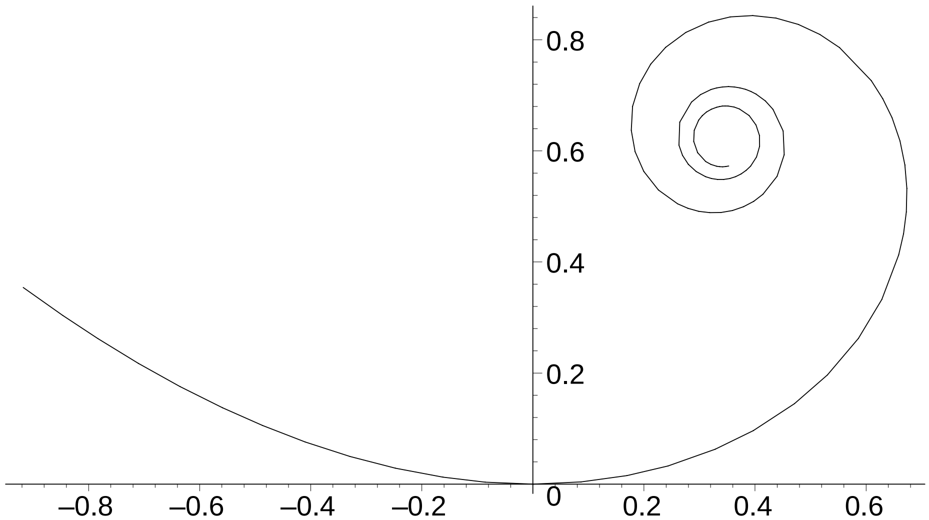
$$k := \exp$$

$$\mathbf{e}^{t} - 1$$

$$\mathrm{Si}(\mathbf{e}^s)\sin(1)+\mathrm{Ci}(\mathbf{e}^s)\cos(1)-\mathrm{Si}(1)\sin(1)-\mathrm{Ci}(1)\cos(1)$$

$$\mathrm{Si}(\mathbf{e}^s)\cos(1)-\mathrm{Ci}(\mathbf{e}^s)\sin(1)-\mathrm{Si}(1)\cos(1)+\mathrm{Ci}(1)\sin(1)$$

```
> plot([F(s),G(s),s=-1..3], color=black);
```



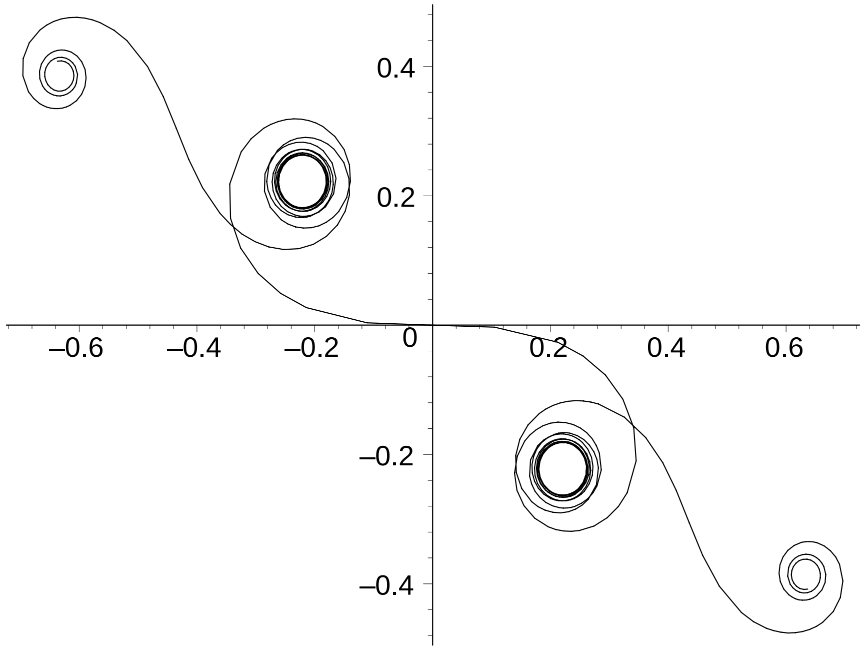

```
> k:=s->s*(s-4)*(s+4);
```

$$k := s \rightarrow s\,(s-4)\,(s+4)$$

```
> [F(s),G(s)];
```

$$\left[\int_0^s \cos\left(\frac{1}{4}\,t^4-8\,t^2\right)dt,\ \int_0^s \sin\left(\frac{1}{4}\,t^4-8\,t^2\right)dt\right]$$

```
> plot([F(s),G(s),s=-5..5],color=black);
```


0.4
0.2
–0.6
–0.4
–0.2
0
0.2
0.4
0.6
–0.2
–0.4

>