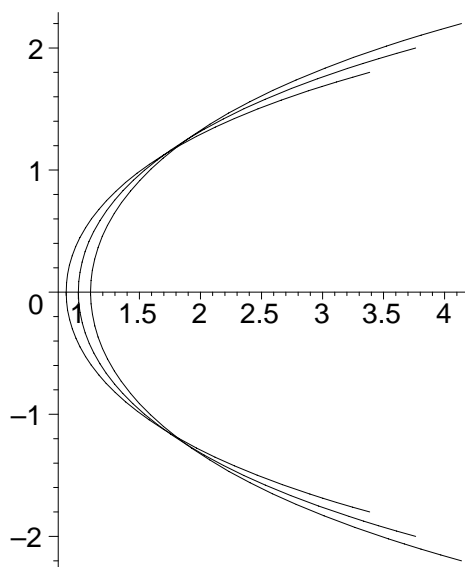


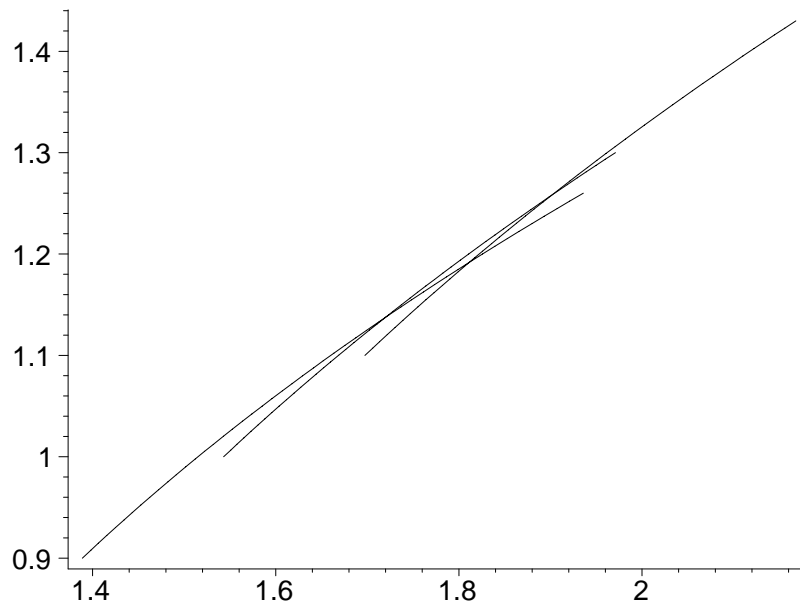
Math 4530  
Monday March 26, 2001  
Stability of the catenoid.

These pictures are related to the formula for the second variation of area, but reasoning is omitted. The variations indicated by the first picture (from the point where the variation is zero) indicate a direction of instability, i.e. a way for the surface to decrease its surface area. This causes the critical catenoid to pop!

```
[ >  
[ >  
[ > with(plots):  
[ > plot1:=plot([cosh(t),t,t=-2..2],color=black):  
[ > plot2:=plot([.9*cosh(t),.9*t,t=-2..2],color=black):  
[ > plot3:=plot([1.1*cosh(t),1.1*t,t=-2..2],color=black):  
[ > display({plot1,plot2,plot3});
```



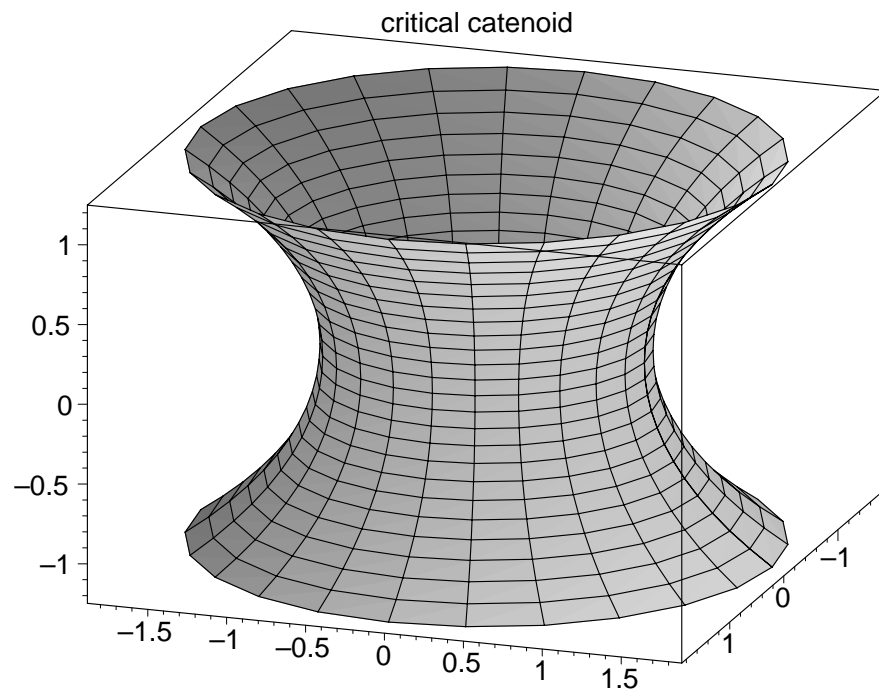
```
[ > plot1:=plot([cosh(t),t,t=1..1.3],color=black):  
[ > plot2:=plot([.9*cosh(t),.9*t,t=1..1.4],color=black):  
[ > plot3:=plot([1.1*cosh(t),1.1*t,t=1..1.3],color=black):  
[ > display({plot1,plot2,plot3});
```



```

> solve(cosh(t)=t*sinh(t),t);
  #this gives the point where the surfaces infinitesimally
  coincide
  #(reasoning omitted)
                                RootOf(_Z(e-Z)2 - _Z - (e-Z)2 - 1)
> tcrit:=evalf(%);
                                tcrit := 1.199678640
> cosh(tcrit)/tcrit;
tcrit/cosh(tcrit);
  #the number in the Oprea book, after problem 3.4 page 139
                                1.508879561
                                .6627434195
> plot3d([cosh(t)*sin(theta),cosh(t)*cos(theta),t],t=-tcrit..tcrit,
theta=0..2*Pi,title="critical catenoid");
  #for fixed circular radius at the top and bottom
  #this is the "tallest" catenoid you can make before
  #it pops into two disks.

```



```
> plot3d([cosh(t)*sin(theta), cosh(t)*cos(theta), t], t=-2..2,  
theta=0..2*Pi, title="post-critical catenoid");  
#you won't see this fellow sitting still
```

