

Math 4200-001
Week 10-11 concepts and homework
3.2-3.3
Due Friday November 6 at 11:59 p.m.

3.3 1ab, 4, 6, 8, 13, 15, 17, 18, 19, 20

w10.1a) Use the definitions of even and odd functions to show that if f is analytic in a neighborhood of the origin and if f is even, then its Taylor series at $z_0 = 0$ only contains even powers of z ; and if f is odd the Taylor series only contains odd powers.

w10.1b) Are the same facts true for the Laurent series based at $z_0 = 0$, for even and odd analytic functions defined in annuli concentric to the origin?

w10.2) Let f be an entire function. Suppose $f\left(\frac{1}{n}\right) = \frac{1}{n^2}$ for all positive integers n .

Is it possible for $f(-1)$ to equal -1 ? Explain.

w10.3) Use power series or L'Hopital's rule to find

$$\lim_{z \rightarrow 0} \frac{\cos(z) - 1}{z \sin(z)}$$

w10.4) Continuing the text problem 3.3.4, find the Laurent series for

$$\frac{1}{z(z-1)(z-2)}$$

valid for $|z| > 2$.

w10.4) Which of these functions has a removable singularity at $z = 0$?

a) $\frac{\cos(z) - 1}{z \sin(z)}$ (see w10.2)

b) $\frac{\cos(z) - 1}{z^3}$.