Math 4200
Monday September 30 review notes

Announcements: We'll finish the proofs in Friday's notes first: homotopy lemma, antiderivatives in simply connected domains; deformation theorem. Then we'll go over today's review notes.

Review session today (Monday) 5:00-6:30 in JTB 120. We'll go over the exam from 2011.

If you're pressed for time, you can hand in the section 2.3 homework for this week on Friday instead of Wednesday, although the exam will likely have some material from 2.3.

Exam Wednesday October 5

begin at 11:45 (5 minutes early), and end at 12:45 (5 minutes late), so that you have an hour for the exam.

closed book and closed notes

Potential Topics (we'll discuss):

Complex differentiability (def at a point, equivalent approximation formula, and "analytic" on a domain).

Cauchy Riemann equations

relationship to real differentiability, i.e. $f : \mathbb{C} \to \mathbb{C}$ analytic at z_0 is equivalent to what for $F : \mathbb{R}^2 \to \mathbb{R}^2$ at (x_0, y_0)?

consequences of definition of derivative and equivalent approximation formula

sum, product, quotient rules
chain rule
chain rule for curves
differential map df_{z_0}
using chain rule for curves to write CR in different coordinate systems.

inverse function theorem

harmonic functions and harmonic conjugates in simply-connected domains

Complex transformations

polar form for complex multiplication, powers, exponentials, logarithms.

$f(z) = az + b, z^n,$ $e^z, \log z,$ $z^a,$ $\cos z, \sin z,$ compositions

branch points, branch cuts, branch domains for root functions, logarithms, and compositions

Contour integration

definition and computation

relation to real-variables line integrals

Green's Theorem for contour integrals around domains (including domains with holes).

contour replacement for C^1 analytic integrands $f(z)$, via Green's Theorem and CR equations (Section 2.2 Cauchy's Theorem)

estimates for modulus of contour integrals.

FTC

evaluation of contour integrals when the integrand is analytic, using FTC and/or contour replacement.

Homotopy-related ideas

homotopies

fixed endpoint

of closed paths

simply-connected domains

Antiderivatives of analytic functions

equivalence to path independence

local anti-derivative theorem, using rectangle lemma

global antiderivatives in (open) simply-connected domains, using homotopy lemma to prove path independence

Deformation Theorems via the homotopy lemma

for contours with fixed endpoints

for closed curves (section 2.3 Cauchy's Theorem)