Math 4200-001
Week 4 concepts and homework
1.5 - 1.6
Due Wednesday September 18 at start of class.

1.5 postponed from last week: 8, 16. New this week: 25, 26, 27, 28, 31.
1.6 1c, 2abc, 3a, 4, 5, 6, 10, 14.

extra credit (5 points) As we discuss in class on Wed. Sept 11, a real-differentiable map $F: A \subseteq \mathbb{R}^2 \to \mathbb{R}^2$ which is conformal and preserves signed angles between tangent vectors can only arise if F corresponds to an analytic function $f: A \subseteq \mathbb{C} \to \mathbb{C}$. Prove this.

Hint: Recall the definition of the differential map from tangent vectors at $(x_0, y_0) \in A$ to tangent vectors at $F(x_0, y_0)$. For each tangent vector $\vec{v} \in T_{(x_0, y_0)}\mathbb{R}^2$, and writing $F(x, y) = (u(x, y), v(x, y))$, we compute the differential map via

$$dF_{(x_0, y_0)}(\vec{v}) = DF(x_0, y_0)\,\vec{v} = \begin{bmatrix} u_x(x_0, y_0) & u_y(x_0, y_0) \\ v_x(x_0, y_0) & v_y(x_0, y_0) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}.$$

Your job is to show that if the differential map preserves oriented angles, then it must be a rotation dilation matrix. A good way to get started is to note that

$$\angle\, dF(\vec{v}), dF(\vec{w}) = \angle\, \vec{v}, \vec{w} \qquad \forall\, \vec{v}, \vec{w} \in T_{(x_0, y_0)}\mathbb{R}^2$$

implies that the two columns of the derivative matrix must perpendicular, by the choice $\vec{v} = [1, 0]^T, \vec{w} = [0, 1]^T$. Also make use of the dot product formula you know for (unoriented) angles, for at least one other choice of $\vec{v}, \vec{w}$,

$$\cos(\theta) = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\|\ \|\vec{w}\|}.$$