Math 4200
Mon 11/14

Review for exam, which will cover 2.4–2.5; 3.1–3.3; 4.1–4.2.
Exam on Wednesday begins 11:45 (5 minutes early)
ends 12:45 (5 minutes late.)

As with 1st exam you will be asked to complete 3 substantial problems
(maybe some basic questions at the start, as last time.)
There will be a mixture of Theorem proofs/explanations and computations.

Bring a copy of the 2007 exam to class (linked at our lecture page), and
we can go over any topics/problems suggested by this review or that exam.

§2.4 Cauchy Integral Formula
- index $I(\gamma; z_0)$
- C.I.F. for γ contractible in domain of $f(z)$ analytic
  - special case if $\gamma = \partial\Omega$ (even with several oriented curves)

- formulas, estimates for derivatives
- Liouville's Thm
- Fund. Thm. Alg.
- Morera's Thm.

2.5 Max. modulus principle & harmonic fcns
- Mean value property for $f(z)$ analytic
- Clever proof for harmonic conjugates in simply connected domains
- Mean value property for harmonic fcns
- Max modulus principle for $f(z)$ analytic; max principle for u harmonic.

3.1 Convergent sequences & series of analytic fcns
- uniform limits of analytic fcns are analytic, and deriv of limit is limit of derivs
- Weierstrass M test

3.2 Power series & Taylors Thm
- radius of conv.
- term by term diff
- uniqueness
- analytic $\Longleftrightarrow$ power series
- multiplication of power series
- examples

3.3 Laurent series
- analytic in annulus (or punctured disk) iff Laurent
- uniqueness
- isolated singularities
- residue
- multiplication of Laurent series

§ 4.1 Calculating residues at isolated singularities

$$f(z) = \frac{f_1(z)}{(z-z_0)^k} + f_2(z)$$

$$f(z) = \frac{g(z)}{h(z)} = \frac{\sum_{n=M}^{\infty} a_n (z-z_0)^n}{\sum_{n=N}^{\infty} \tilde{a}_n (z-z_0)^n} = \frac{(z-z_0)^M}{(z-z_0)^N} \phi(z) \qquad \phi(z) \text{ analytic } \phi(z_0) \neq 0.$$

simple poles

table if desparation strikes

§ 4.2 Residue theorem

statement & proof for γ contractible in A (via deformation thm & Cauchy thm)

statement & proof if $\gamma = \partial\Omega$, possibly with several component curves (via Green's thm).

contour integral computations via residue thm

residues at ∞.