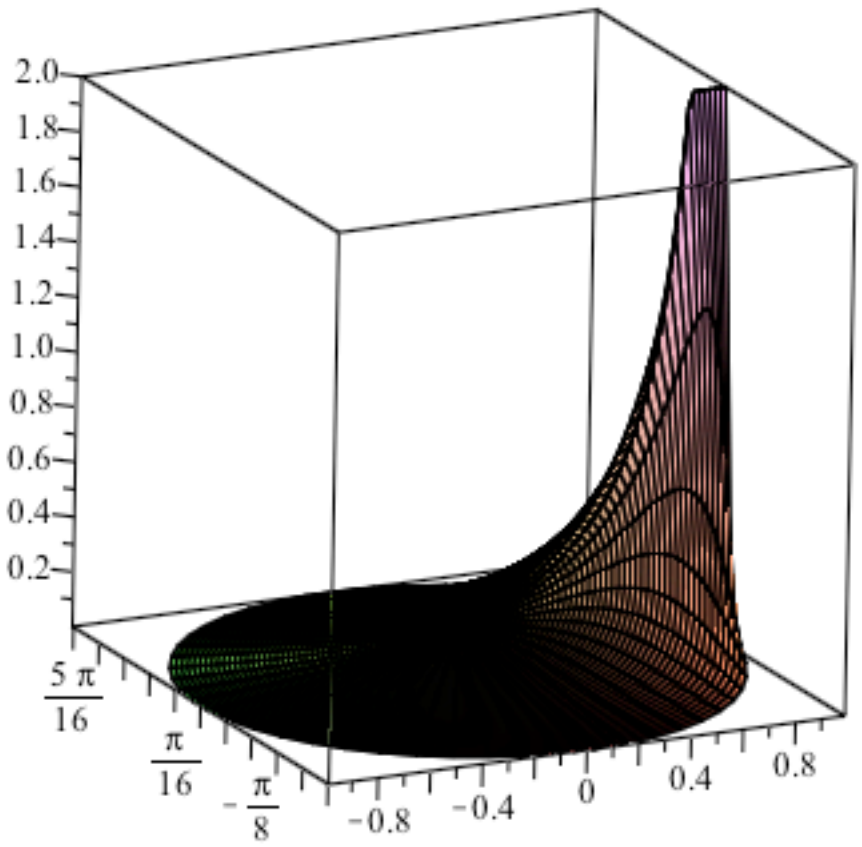
Poisson Integral formula for harmonic functions

```
> pkernel := (rho, phi) -> 1/(2*Pi) * (1 - rho^2)/((1 - rho*cos(phi))^2 + rho^2*sin(phi)^2);
```

$$pkernel := (\rho, \phi) \rightarrow \frac{1}{2} \frac{1 - \rho^2}{\pi \left((1 - \rho \cos(\phi))^2 + \rho^2 \sin(\phi)^2 \right)} \quad (1)$$

```
> with(plots):
> plot3d([rho*cos(phi), rho*sin(phi), min(pkernel(rho, phi), 2)], rho = 0..(.99), phi = 0..2*Pi,
      axes = boxed, grid = [50, 50], title = `delta function heat source`);
```

delta function heat source

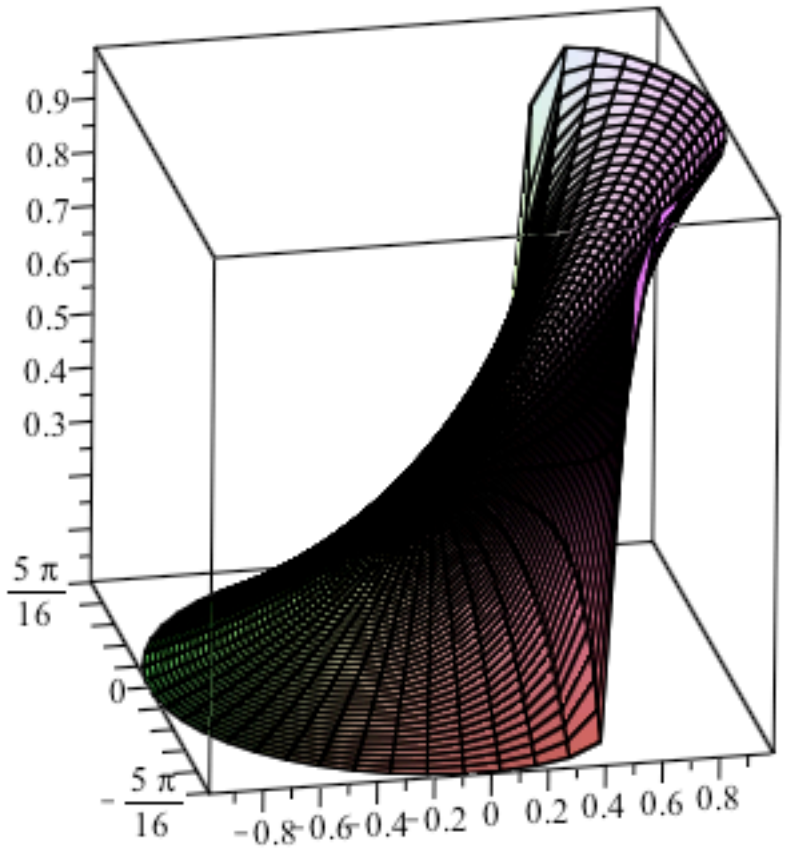



```
> hotcold := (rho, phi) -> int(pkernel(rho, phi - theta), theta = -Pi/3 .. Pi/3.0);
```

$$hotcold := (\rho, \phi) \rightarrow \int_{-\frac{1}{3}\pi}^{\frac{\pi}{3.0}} pkernel(\rho, \phi - \theta)\, d\theta \quad (2)$$

```
> plot3d( [ρ·cos(ϕ), ρ·sin(ϕ), hotcold(ρ, ϕ) ], ρ = 0 .. (.99), ϕ = 0 ..2·π, axes = boxed, grid
      = [50, 50 ], title = `hot and cold boundary temperatures` );
```

hot and cold boundary temperatures




```
>
```