**Math 4200**

**Fri 7 Sept**

- chain rules, page 4 Wed.

- then skip back to this page to visualize the differential map in an example.

\[
f(z) = z^2 \quad f'(z) = 2z
\]

\[
z_0 = 1 + i \quad f(z_0) = (1+i)^2 = 2i
\]

If \(y(0) = 1 + i\),

\[
\frac{d}{dt} f(y(t)) = f'(1+i) y'(t) \quad t=0
\]

so \(df\) at \(1+i\)

\[
= 2\sqrt{2} e^{i\pi/4} y'(0)
\]

so \(df\) is rotation and dilation, with

\[
\text{rot} = \pi/4, \quad \text{stretch} = 2\sqrt{2}
\]

**HW for Fri Sept 14**

- 5.1.5 5c, 6b (for 5c, 6b, draw L-square diagram to illustrate the differential map)

- 19, 25, 26, 28, 31

Class exercise (i): today's notes

- 5.1.6 1c, 2abc, 3a, 4, 6, 10, 14

[Diagram showing function mapping and differential map visualization]
You needed this application of the chain rule for curves to do one of your HW problems for today. (I guess I assumed you already knew the real-variables version.)

**Def.** A ⊆ ℂ is path connected iff

∀ P, Q ∈ A ∃ C¹ curve γ : I = [a, b] → A

\[ γ(0) = P \]
\[ γ(b) = Q \]

**Theorem.** If A is open and path connected (b.t.w. this is implied by the hypothesis that A is open and connected) and \( f : A \rightarrow ℂ \) is analytic with \( f'(z) \equiv 0 \)

then \( f \) is constant.

**Proof.** Let \( P \in A \). We will show \( f(z) = f(P) \), a const, \( \forall z \in A \)

Let \( Q \in A \)

Let \( γ : [a, b] \rightarrow A, γ \in C¹, γ(a) = P, γ(b) = Q \)

(1) \[ f(Q) - f(P) = \int_a^b γ'(t) \, dt \]

by FTC (on real & imag parts)

(2) \[ = \int_a^b f'(γ(t)) γ'(t) \, dt \]

(chain rule for curves)

= \[ \int_a^b 0 \, dt = 0 \]

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**Class exercise:** If \( f'(z₀) = a + bi \) then the real form of the differential map

\[ dF_{(z₀, y₀)} (x', y') = \left( (F \circ γ)'(t₀) \right) \]

Analytic maps are called **conformal** whenever \( f'(z₀) \neq 0 \), because the differential map is a rotation-dilation, so "preserves shape."

Prove that you only need to preserve angles and orientation to also get uniform scaling. That is

\[ \det [dF] > 0 \]

and \( \det [dF] > 0 \)