Math 4200

~~Fri 7 Sept~~

~~HW for Fri Sept 14~~

§1.5 5c, 6b (for 5c, 6b, draw L-square diagram to illustrate the differential map)

19, 25, 26, 28, 31

Class exercise (1): today's notes

§1.6 1c, 2abc, 3a, 4, 6, 10, 14

- chain rules page 4 Wed.

- then skip back to this page to visualize the differential map in an example

$f(z) = z^2 \qquad f'(z) = 2z$

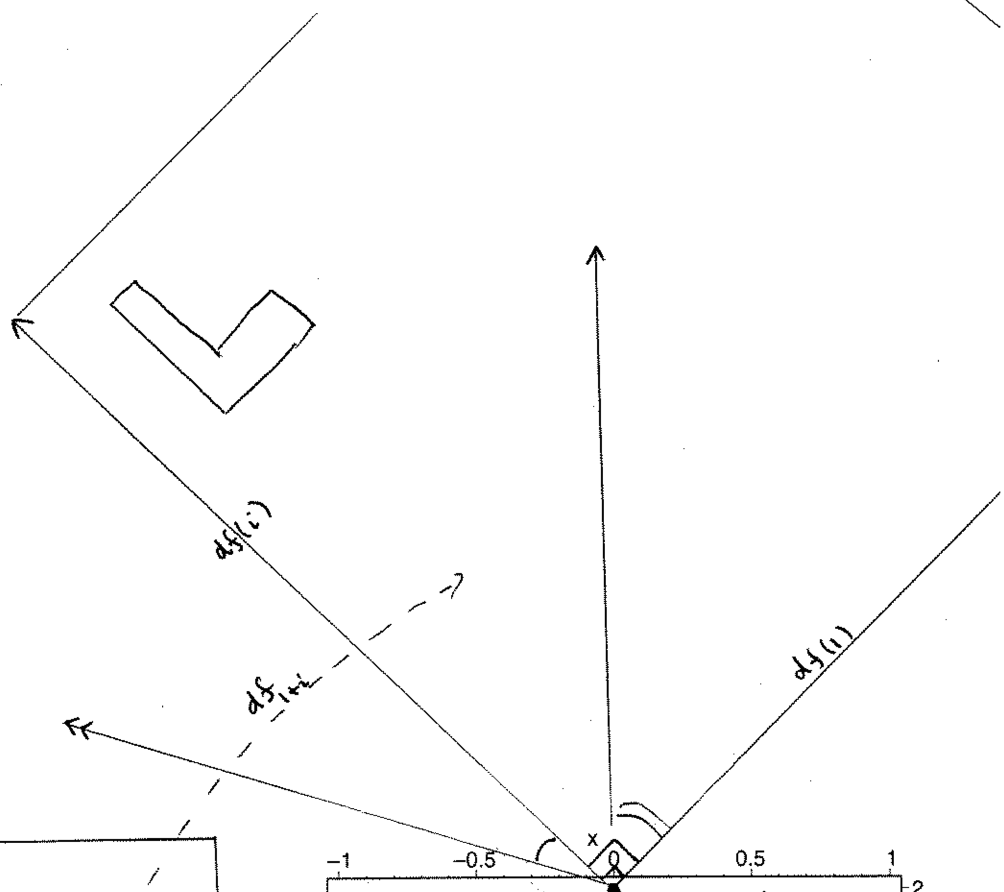
$z_0 = 1+i \ , \ f(z_0) = (1+i)^2 = 2i$

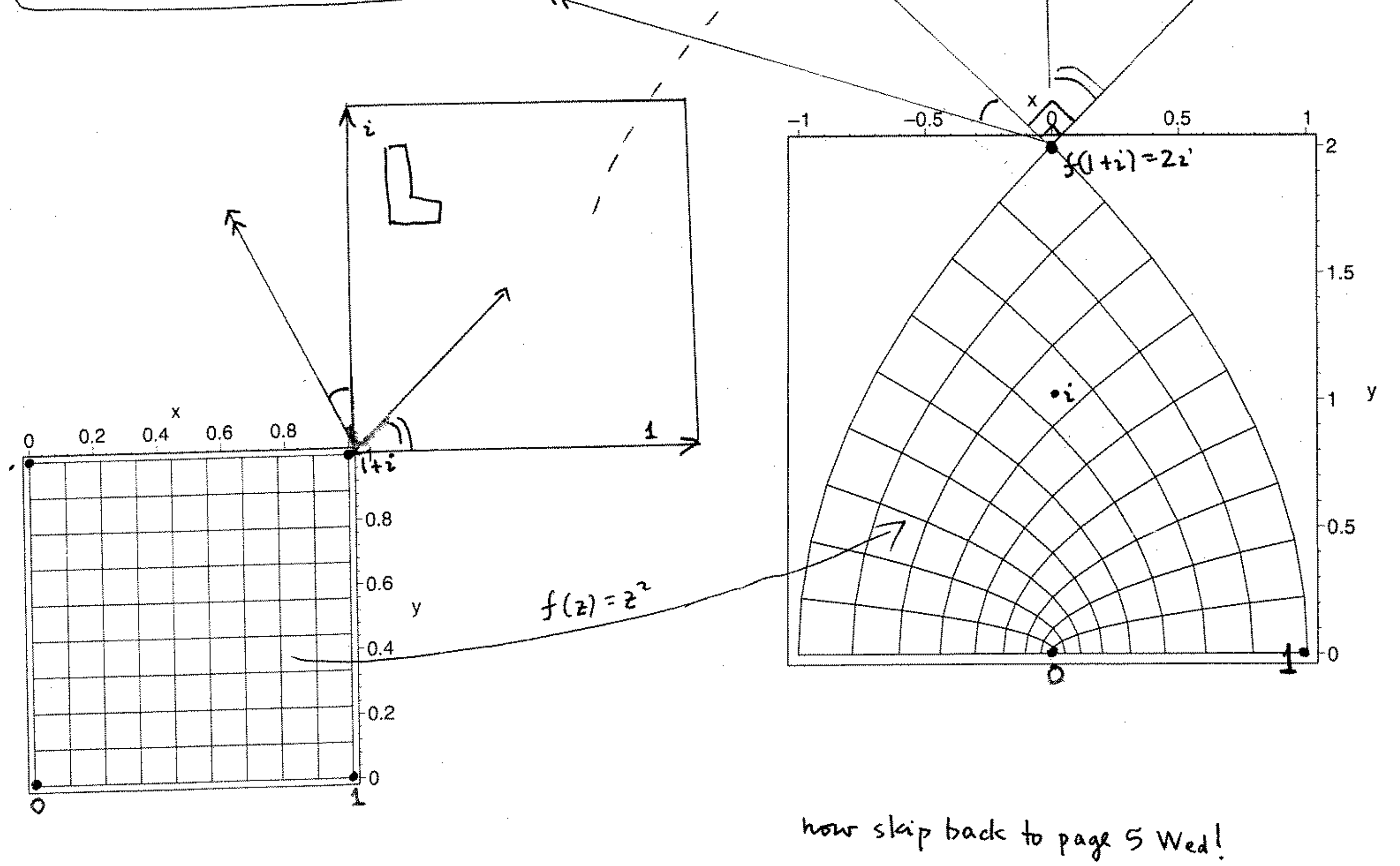
If $\gamma(0) = 1+i$

$$\frac{d}{dt} f(\gamma(t))\Big|_{t=0} = f'(1+i)\,\gamma'(0)$$

so $df_{1+i}(\gamma'(0)) = 2(1+i)\gamma'(0)$
$\qquad = 2\sqrt{2}\, e^{i\pi/4}\gamma'(0)$

so df_{1+i} is rotation dilation,
with rot $= \pi/4$
stretch $= 2\sqrt{2}$





now skip back to page 5 Wed!

You needed this application of the chain rule for curves to do one of your HW problems for today. (I guess I assumed you already knew the real-variables version.)

<u>Def</u>: $A \subset \mathbb{C}$ is <u>path connected</u> iff
$\forall\, P, Q \in A \quad \exists\, C^1$ curve $\gamma : \underset{\mathbb{R}}{\underset{\cap}{I}} = [a,b] \to A$
$\gamma(a) = P$
$\gamma(b) = Q$

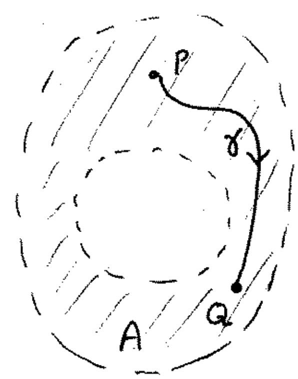
<u>Theorem</u> If A is open and path connected (b.t.w this is implied by the hypothesis that A is open and connected) and $f : A \to \mathbb{C}$ is analytic with $f'(z) \equiv 0$ then f is constant.

proof: Let $P \in A$. We will show $f(z) \equiv f(P)$, a const, $\forall z \in A$

<u>pf</u> Let $Q \in A$
Let $\gamma : [a,b] \to A$, $\gamma \in C^1$, $\gamma(a) = P$, $\gamma(b) = Q$

(*) $\quad f(Q) - f(P) = \displaystyle\int_a^b \frac{d}{dt}\left(f(\gamma(t))\right) dt$

by $\underset{1210}{\text{FTC}}$ (on real & imag parts)



$(*) = \displaystyle\int_a^b f'(\gamma(t))\,\gamma'(t)\,dt$

(chain rule for curves)

$= \displaystyle\int_a^b 0\,dt = 0 \quad ∎$

<u>Class exercise</u> ①: If $f'(z_0) = a + bi$ then the real form of the differential map

$$dF_{(x_0,y_0)}(\gamma'(t_0)) := (F \circ \gamma)'(t_0) = \begin{bmatrix} u_x & u_y \\ v_x & v_y \end{bmatrix} \begin{bmatrix} x'(t_0) \\ y'(t_0) \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Analytic maps are called <u>conformal</u> wherever $f'(z_0) \neq 0$, because the differential map is a rotation-dilation, so "preserves shape."

Prove that you only need to preserve angles and orientation to also get uniform scaling.
That is

$$\left.\begin{array}{l} \measuredangle\, dF(\vec{v}, \vec{w}) = \measuredangle\, \vec{v}, \vec{w} \\ \text{and } \det[dF] > 0 \end{array}\right\} \Rightarrow dF \text{ is a rotation-dilation}$$