Math 4200
Fri 21 Sept

HW for Fri 28 Sept:
add §2.3 to the earlier §2.1-2.2 HW

2.3 1, 3, 4, 5, 6, 7, 9, 10.

We are finishing (from Monday notes)

<u>Theorem 1</u> $\underset{\mathbb{C}}{\overset{A}{\cap}}$ open, connected, $f: A \to \mathbb{C}$ continuous.

Then the following are equivalent:

(i) $\exists\, F: A \to \mathbb{C}$ s.t. $F'(z) = f(z)\ \forall z \in A$ (and F is unique up to a constant).

(ii) $\forall$ choices of initial pt $P \in A$ and terminal pt $Q \in A$

$$\int_{\gamma_0} f(z)\,dz = \int_{\gamma_1} f(z)\,dz$$

whenever γ_0, γ_1 are p.w. C^1, start at P and end at Q.
("path independence")

(iii) $\forall$ p.w. C^1 γ with are <u>closed</u> (i.e. initial pt = terminal pt),

$$\int_\gamma f(z)\,dz = 0$$

(i) $\Rightarrow$ (iii) $\Rightarrow$ (ii) done last time.

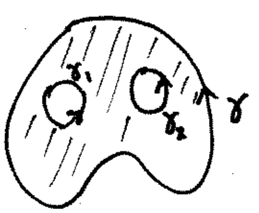
(ii) $\Rightarrow$ (i):

<u>Theorem 2</u> If A is <u>open</u> and <u>simply connected</u>, $f: A \to \mathbb{C}$ analytic and C^1
then $\exists\ F: A \to \mathbb{C}$ s.t. $F'(z) = f(z)\ \forall z \in A$

pf: Green's Thm! (which is itself explained on page 2 Wed notes)

Note, while proving Theorem 2, and appealing ~~th~~ to Green's Theorem (for domains with holes), we actually proved:

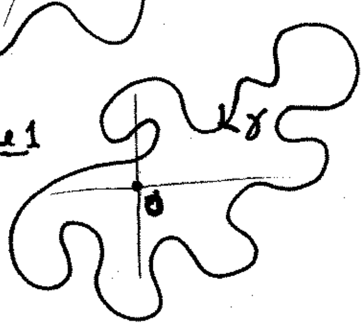
<u>Theorem</u> Let $f(z)$ be analytic and C^1 on ~~asso~~ an open region containing a compact subset A, whose boundary is a union of piecewise C^1 curves. Let γ be the outer boundary curve, with $\gamma_1, \ldots$ the inner bdry curves. Orient curves as indicated



Then $\displaystyle\int_\gamma f(z)\,dz = \sum_i \int_{\gamma_i} f(z)\,dz$

(notice we changed the orientation of the γ_i from Greens)

<u>Example 1</u>



$$\int_\gamma \frac{1}{z}\,dz =$$

<u>Example 2</u> Let $\gamma = \{z \mid |z| = 2\}$. (Oriented c.c.)

What is $\displaystyle\int_\gamma \frac{1}{z^3 - z}\, dz$?

note, $\frac{1}{z^3-z} = -\frac{1}{z} + \frac{1/2}{z-1} + \frac{1/2}{z+1}$