Math 4200
Friday Oct 20

Homework for Friday Oct 26

2.5   2, 5, 8, 9a, 10, 15, 18
3.1   4, 6, 7, 14
3.2   2, 3, 4, 7

Recall the mean value property for f analytic on A, $\overline{D(z_0;R)} \subset A$:

$$f(z_0) = \frac{1}{2\pi}\int_0^{2\pi} f(z_0 + Re^{i\theta})\,d\theta$$

<u>Exercise 1</u>: Use a limiting argument to show that if f is continuous on $\overline{D(z_0;R)}$ and analytic inside $D(z_0;R)$, then the mean value property still holds
(more practice interchanging limits and integrals)

<u>Exercise 2</u>: If $u(x,y)$ is harmonic in $B((x_0,y_0),R) = \{(x,y) \mid \|(x-x_0, y-y_0)\| < R\}$, if u is also C^2 in B and continuous on the closed ball $\overline{B}$,
use conjugate function theorem to show that u also satisfies the mean value property

$$u(x_0,y_0) = \frac{1}{2\pi}\int_0^{2\pi} u(x_0 + R\cos\theta,\ y_0 + R\sin\theta)\,d\theta$$

<u>Exercise 3</u> Use the mean value property to prove the amazing (and surprisingly useful theoretical fact)

<u>maximum modulus principle</u> :

Let $A \subset \mathbb{C}$ bounded and open and connected

$f : \bar{A} \to \mathbb{C}$ continuous

$f : A \to \mathbb{C}$ analytic

Then $\max \{ |f(z)| \text{ s.t. } z \in \bar{A} \} = \max \{ |f(z)| \text{ s.t. } z \in \partial A \}$

If $\exists\, z_0 \in A$ s.t. $|f(z_0)|$ is this maximum modulus value, then $f(z) \equiv f(z_0)$ is constant

<u>proof</u>: Since $|f(z)|$ is cont. on $\bar{A}$, $M := \max \{ |f(z)| \text{ s.t. } z \in \bar{A} \}$ exists (and is finite)

It suffices to show that if $\exists\, z_0 \in A$ s.t. $|f(z_0)| = M$, then f is constant!

Let $B = \{ z \in \bar{A} \text{ s.t. } f(z) = f(z_0) \}$. Show B is open and closed in A:

<u>Exercise 4</u> What is $\max \{ |e^z| \text{ s.t. } z \in \overline{D(0;1)} \}$?

Exercise 5: Suppose two analytic functions on A are continuous on $\bar{A}$, and have the same values on ∂A.
What can you say about the two functions? (A is bounded)

Exercise 6: If u is harmonic (and C^2) on A and continuous on $\bar{A}$, what can you say about $M = \max\{u(z), z \in \bar{A}\}$
$m = \min\{u(z), z \in \bar{A}\}$?