Math 4200
Wed Oct 17

Finish careful proof of fundamental theorem of algebra, using Liouville's Thm.
Then Morera.

More consequences & estimates:

Exercise 1 Let $f(z)$ be analytic in A open, $\overline{D(0;R)} \subset A$ ($z_0 = 0$ is chosen for convenience).
Assume $|f(z)| \le M$ $\forall z \in \overline{D(0;R)}$.

Use CIF, and derivative formulas to make best possible estimates for

$f'(z)$
$f^{(n)}(z)$, $z \in D(0;R)$

(These estimates will be useful in HW.)

Exercise 2 (Mean value property)

$$f : \underset{\substack{\cup \\ \overline{D(z_0;R)}}}{A} \to \mathbb{C} \text{ analytic}$$

(a) Prove $f(z_0) = \frac{1}{2\pi}\int_0^{2\pi} f(z_0 + Re^{i\theta})\, d\theta$, the mean value property for analytic functions.

(b) Deduce that if $u(x,y)$ is a harmonic function and C^2, on an open set containing $\{(x,y) \mid \|(x,y)-(x_0,y_0)\| \le R\}$

then

$$u(x_0,y_0) = \frac{1}{2\pi}\int_0^{2\pi} u((x_0,y_0) + R(\cos\theta, \sin\theta))\, d\theta$$

the mean value property for harmonic functions

"Exercise" 3 Use the Mean value property to prove the (amazing?) maximum modulus principle:

If $f : A \to \mathbb{C}$ analytic

A open and connected. (and bounded)

Assume f extends as a continuous function to $f : \overline{A} \to \mathbb{C}$

Then $\max\{|f(z)| \text{ s.t. } z \in \overline{A}\} = \max\{|f(z)| \text{ s.t. } z \in \partial A\}$

In fact, if $\exists\, z_0 \in A$ for which $|f(z_0)|$ is this const. value, then $f(z)$ is constant.

(probably we'll just do an example of this today)