Math 4200
Friday Nov 2

projects?

HW for Nov. 9
3.3 1ab, 2, 4, 6, 8, 9, 13, 15, 17, 18, 19, 20
4.1 1de, 3, 5, 7ab, 9

- Finish the amazing cloning theorems, Wed notes.

<u>Corollary</u> Let $f: A \to \mathbb{C}$ analytic, $z_0 \in A$

Let $R_1 := \sup\{r>0 \text{ s.t. } D(z_0,r) \subset A\}$.

Let $f(z) = \sum_{n=0}^{\infty} a_n (z-z_0)^n$ be the Taylor series for f at z_0.

Then its radius of convergence $R \geq R_1$.

If $f(z)$ has no continuous extension to any $D(z_0; R_1+\varepsilon)$, for any $\varepsilon>0$, then $R = R_1$

<u>Example</u> $f(z) = \tan z = \frac{\sin z}{\cos z}$

$z_0 = 0$.

$\tan z$ is defined on $\mathbb{C} \setminus \{z \mid \cos z = 0\}$.

and $|\tan z| \to \infty$ as $z \to \frac{\pi}{2} + \pi k \quad (k \in \mathbb{Z})$.

Thus the radius of convergence for

$$\tan z = \sum_{n=0}^{\infty} a_n z^n \qquad a_n = \frac{f^{(n)}(0)}{n!}$$

<u>is</u> $R = \pi/2$!

$$\cos z = \cos(x+iy)$$
$$= \cos x \cos iy - \sin x \sin iy$$
$$= \cos x \left(\tfrac{1}{2}(e^{-y}+e^{y})\right) - \sin x \left(\tfrac{1}{2i}\right)(e^{-y}-e^{y})$$

$$\cos z = 0 \iff \begin{cases} \cos x = 0 \\ y = 0 \end{cases}$$
$$\iff z = \tfrac{\pi}{2} + \pi k \quad k \in \mathbb{Z}.$$

<u>proof of Corollary</u>: Write $g(z) = \sum_{n=0}^{\infty} a_n (z-z_0)^n$.

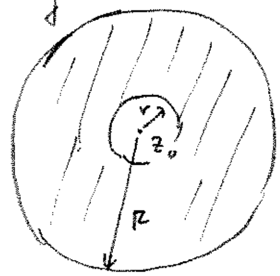
If $R > R_1$ then g is analytic in $D(z_0; R)$ and $g = f$ in $D(z_0; R_1)$

Thus f <u>does</u> have a continuous extension to some $D(z_0; R_1+\varepsilon)$

$\Rightarrow\!\Leftarrow$. ∎

↳ 3.3 Laurent series.

for $f(z)$ analytic in an annulus $\{ z \mid r < |z - z_0| < R \}$



on Monday,
we will use C.I.F. and clever use of geometric series to show $f(z)$ has unique series rep. (called the <u>Laurent series</u>)

$$f(z) = \sum_{n=0}^{\infty} a_n (z-z_0)^n + \sum_{n=1}^{\infty} b_n / (z-z_0)^n$$

conv for $|z-z_0| < R$
(unif. abs. for $|z-z_0| \le R - \varepsilon$)

conv. for $|z-z_0| > r$
(unif. abs for $|z-z_0| \geq r + \varepsilon$)

<u>Example</u>

Let $f(z) = \dfrac{1}{(z-1)(z-2)}$.

Find

a) Taylor series @ $z_0 = 0$
b) Laurent series for $1 < |z| < 2$
c) Laurent series for $|z| > 2$.

<u>Hint</u>: You can do it all with partial fractions and geometric series