Interesting geometric fact: the level curves of harmonic conjugates are always $L$. There are 2 good explanations of this fact:

Example: $f(z) = z^2$

$u = x^2 - y^2, \quad v = 2xy$
Differentiation of basic functions

\[ e^z \rightarrow \frac{d}{dz} e^z = e^z \quad (\text{HW}) \]

\[ \cos z \]

\[ \sin z \]

\[ \cosh z \]

\[ \sinh z \]

\[ \tan z \]

etc.

\[ \log z = \ln |z| + i \arg z \]

principal branch

\[-\pi < \arg z < \pi \]

(could use other branches)

\[ \frac{d}{dz} \log z = \frac{1}{z} \]

give 2 proofs: (a) lnv fn thm

(b) CR.

\[ z^a = e^{a \log z} \]

\[ \Rightarrow z^{\frac{1}{n}} = ? \]

\[ a^z = e^{z \log a} \]

There are subtle questions related to finding branches of funs, e.g. \( \sqrt[3]{z} \) (in book)

\( \sin^{-1}(z), \cos^{-1}(z) \) (in HW)