Math 4200
Friday 9/30

Exam Monday!
You are responsible for key definitions & theorems;
you may be asked about these explicitly or implicitly.
HW problems & Chapter review problems (Chptr 1 all, Chptr 2 1a, 3, 4, 5, 9, 13, 14, 15, 17, 19, 21)
are good to understand.

Chptr 1

- complex numbers
  - $+, \cdot, \bar{}, |\,|$
  - polar form
  - DeMoivre & Euler
  - finding roots
- elementary fcns
  - e^z, $z^{1/n}$
  - $\cos z, \sin z, \cosh z, \ldots$
  - $\log z$
- complex differentiable
  - def
  - real interpretation
  - affine approximations
  - CR
- differentiation rules
  - standard ones
  - chain rules (2!)
  - conformal
- Inverse Fcn Thm
- branches of multivalued fcns.

Chptr 2

- contour integrals
  - def — real form
  - properties,
  - $\gamma_1 + \gamma_2$, $-\gamma$
  - estimates (Δ ineq stuff)
- FTC
- path independence
- homotopy
- simply connected
- convex, star-shaped
- "rectangle thm"
- deformation thm
- antiderivs in simply connected domains

Connections

complex diffble $\Leftrightarrow$ complex affine approx $\Leftrightarrow$ real affine approx $\Leftarrow$ C^1 with rot-dil deriv matrix

(real affine approx $\supset$ "rotation-dilation" deriv matrix)

A open, connected, $f: A \to \mathbb{C}$

$\exists F$ s.t. $F' = f$ $\Leftrightarrow$ contour integrals are path ind. $\Leftrightarrow$ contour integrals over closed curves are zero

$f: D(z_0, r) \to \mathbb{C}$ analytic $\Rightarrow \exists F$ s.t. $F' = f$

$\Uparrow$

$f: D(z_0, r) \to \mathbb{C}$ cont s.t. $\oint_{\partial R} f\,dz = 0$ $\forall$ closed rectangles in D

$\Uparrow$

$f: D(z_0, r) \to \mathbb{C}$ analytic

$\Longrightarrow$

Deformation Thm:

$f: A \to \mathbb{C}$ analytic

(a) γ_0 homotopic to γ_1 with fixed endpoints $\Rightarrow \int_{\gamma_0} f\,dz = \int_{\gamma_1} f\,dz$

(b) γ_0 homotopic to γ_1 as closed curves $\Rightarrow \int_{\gamma_0} f\,dz = \int_{\gamma_1} f\,dz$

$\Downarrow$

If $f: A \to \mathbb{C}$ analytic, A simply connected, then $\int_\gamma f\,dz = 0$ $\forall$ closed curves γ

so.. $\exists F$ s.t. $F' = f$ contour integrals are path ind.

In §2.2 we proved weaker forms of Def Thm's (a), (b), using Green's Thm:

If $\gamma = \partial A$ is the oriented boundary of a region, and if $f: A \to \mathbb{C}$ is analytic and C^1 on $cl(A)$, then

$$\int_\gamma f(z)\,dz = 0$$

can you explain each arrow? (and box)