Math 4200
November 15, 2002
**EXAM 2**

Each complete problem below is worth 25 points. Choose any four out of the six problems to do. If you try 5 or 6 problems, indicate clearly which four you want graded. This exam is closed book and closed note, except for the residue and contour integral tables you requested. Show complete work for complete credit. Justify all steps in theorem proofs. Good Luck!

1a) State and prove the Cauchy Integral Formula, which relates the value of an analytic function at a point z to a certain contour integral.

(20 points)

1b) What is the corresponding formula for the nth derivative of f at z? Explain very briefly how this formula is derived.

(5 points)

2) Prove that if f(z) is analytic in a neighborhood of $z_0$, then f(z) has a power series expansion in some disk centered there. Identify the coefficients in this expansion.

(25 points)

3a) Find the first three non-zero coefficients in the Laurent series for f(z)=cot(z), centered at $z_0$=0.

(20 points)

3b) In what annulus does the (3a) Laurent series converge? Explain.

(5 points)

4a) Let $\gamma$ trace the circle |z|=2 in the counterclockwise direction. Use the Residue Theorem to compute

$$\int_{\gamma} 2\frac{z^2}{z^3-1}\,dz$$

(15 points)

4b) Using residue at infinity, i.e. making the substitution $\zeta = \frac{1}{z}$, recompute the integral in (4a).

(10 points)

5) Use contour integration to compute

$$\int_{-\infty}^{\infty} \frac{1}{x^2 + x + 1}\, dx$$

If you use a contour integration table entry to find the value, you must prove why the table entry is valid. Alternately, you may justify your computations for this particular case.

(25 points)

6) Use the theory developed in section 4.4 to find the value of

$$\sum_{n=-\infty}^{\infty} \frac{1}{\left(n - \frac{1}{2}\right)^2}$$

(25 points)