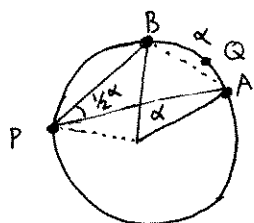


Class exercises: (I was Laplace problem last week)

II. Use trigonometry (mainly the fact that ^{interior} angles of Δ 's add up to π) to show that on the unit circle, if we consider an arc of angle (& length) α , connecting A to B



(a) and if P is any point on the unit circle not on the arc, then

$$\angle APB = \frac{1}{2}\alpha$$

(b) if Q is any point on the arc, then

$$\angle AQB = \pi - \frac{1}{2}\alpha$$

(c) Use (a), (b) to show that the solution to Laplace's eqn $\Delta u = 0$ in unit disk

$$\text{with } \begin{cases} u(e^{i\theta}) = 1, & 0 < \theta < \pi/2 \\ u(e^{i\theta}) = 0 & \pi/2 < \theta < 2\pi \end{cases}$$

$$\text{equals } \left[\text{Im} \left(\log \left(\frac{z-i}{z+i} \right) \right) - \frac{\pi}{4} \right] \frac{3}{4\pi}$$

choose $\frac{\pi}{2}$
 $0 < \arg(z-i) < \pi$
 $-\frac{\pi}{2} < \arg(z+i) < \frac{\pi}{2}$

[this was the $u(e^{i\theta})$ graphed in Maple notes using Poisson integral formula.]

III In PDE's (3150) or 2280 you may have solved the Dirichlet problem for a harmonic function by "separation of variables" in polar coords. Here's what you would end up with:

Let $f(\theta)$ represent the boundary values $u(e^{i\theta})$ for which you want to find a harmonic fcn $u(re^{i\theta})$, $r < 1$.

Let $f(\theta)$ have Fourier series

$$f \sim a_0 + \sum_{n=1}^{\infty} a_n \cos n\theta + \sum_{n=1}^{\infty} b_n \sin n\theta$$

Then (with hand waving),

$$u(re^{i\theta}) \text{ ~~u(r, \theta)~~ } = a_0 + \sum_{n=1}^{\infty} \underbrace{a_n r^n}_{\text{Re}(z^n)} \cos n\theta + \sum_{n=1}^{\infty} \underbrace{b_n r^n}_{\text{Im}(z^n)} \sin n\theta$$

was produced as the solution to $\begin{cases} \Delta u = 0 & |z| < 1 \\ u(e^{i\theta}) = f(e^{i\theta}) & \text{on } \partial D \end{cases}$

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(\theta) d\theta \quad (\text{av. val.})$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cos n\theta d\theta \quad n \geq 1$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \sin n\theta d\theta \quad n \geq 1$$

"Fourier coeffs".

From pages 1-2 on Monday 11/21 notes, we produced the Poisson Integral formula:

$$u(re^{i\theta}) = \frac{1}{2\pi} \int_0^{2\pi} u(e^{i\phi}) \left[\frac{1}{1-z\bar{e}^{i\phi}} + \frac{\bar{z}}{1-\bar{z}e^{i\phi}} \right] d\phi$$

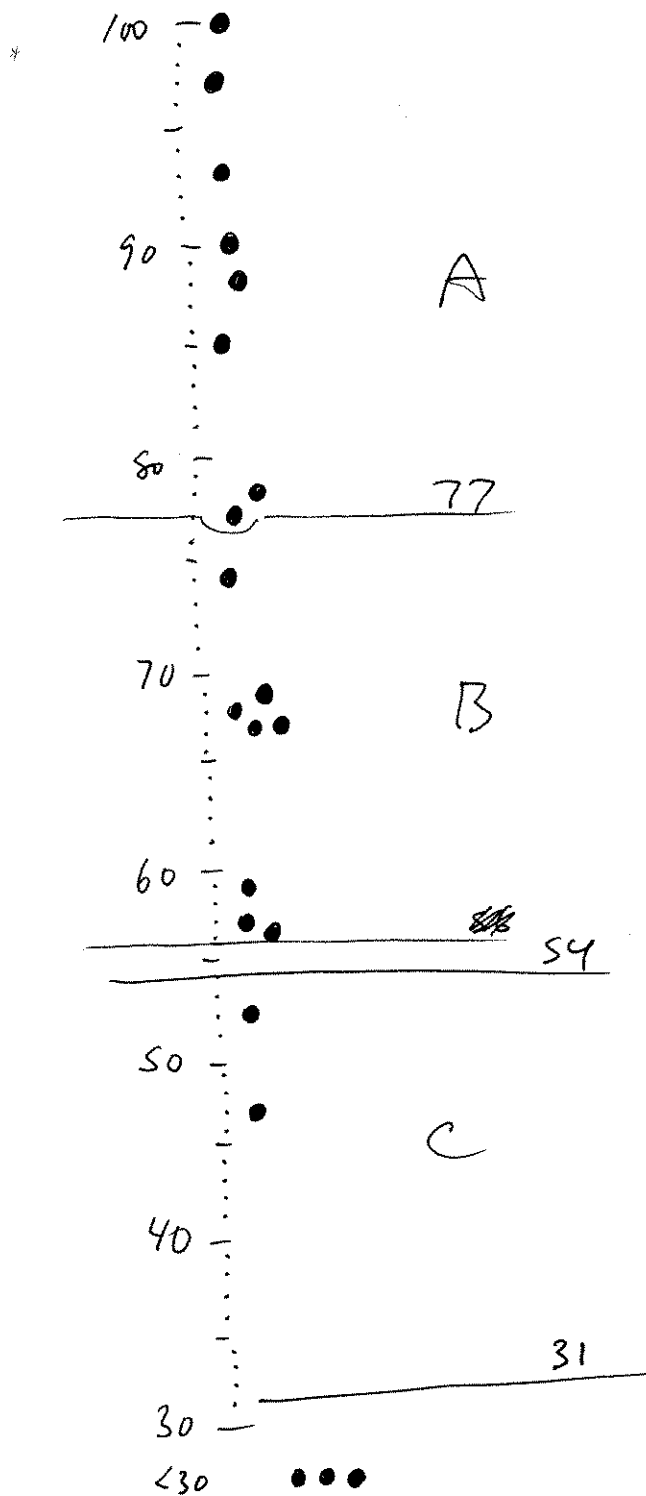
$\uparrow f(\theta)$ in this case

$$z = re^{i\theta}, |z| < 1, \quad \bar{z} = e^{-i\theta}$$

SHOW this produces the separation of variables soln (except using $e^{i\theta}$ rather than r, θ)

Math 4200

Exam 2 score distribution



(exam 1 cutoffs were
78/56/34)