Math 4200 Fri Nov 11

Laplace transform for linear DE's (or systems of DE's)

Example

Exam Wednesday next week
Bobby Hanson will lead
a problem session
Monday 1-3, JTB 120
(here)

Classical resonance problem:

Solve

IVP $\begin{cases} x''(t) + \omega_0^2 x(t) = A \sin \omega_0 t \\ x(0) = x_0 \\ x'(0) = v_0 \end{cases}$

<u>Method 1</u>: Find $x_H(t)$; homogeneous soltn with 2 free parameters

Find some $x_p(t)$; particular soltn.

general soltn $x(t) = x_p(t) + x_H(t)$

Solve matrix eqtn to determine x_H parameters from x_0, v_0.

<u>Method 2</u>: Write $X(s) = \mathcal{L}\{x(t)\}(s)$.

Use Laplace transform table to convert IVP
into algebra eqtn for $X(s)$; find $X(s)$
then use table in reverse direction to write $x(t)$

$$\mathcal{L}(\text{IVP}) \quad \left\{ \; s^2 X(s) - s x_0 - v_0 + \omega_0^2 X(s) = A \frac{\omega_0}{s^2+\omega_0^2} \right.$$

$$X(s)(s^2+\omega_0^2) = A\frac{\omega_0}{s^2+\omega_0^2} + s x_0 + v_0$$

$$X(s) = A\frac{\omega_0}{(s^2+\omega_0^2)^2} + \frac{s x_0}{s^2+\omega_0^2} + \frac{v_0}{s^2+\omega_0^2}$$

$$x(t) = A\omega_0 \frac{1}{2\omega_0^3}(\sin\omega_0 t - \omega_0 t\cos\omega_0 t) + x_0\cos\omega_0 t + \frac{v_0}{\omega_0}\sin\omega_0 t$$

$$x(t) = \frac{A}{2\omega_0^2}(\sin\omega_0 t - \omega_0 t\cos\omega_0 t) + x_0\cos\omega_0 t + \frac{v_0}{\omega_0}\sin\omega_0 t$$

RESONANCE

$$\mathcal{L}\{f'(t)\} = \int_0^\infty e^{-st}\underbrace{f'(t)\,dt}_{dv}$$

$$= f(t)e^{-st}\Big]_{t=0}^{\infty} - \int_0^\infty e^{-st}(-s)f(t)\,dt$$

$$= -f(0) + sF(s)$$

So, $\mathcal{L}\{f''(t)\}(s) = -f'(0) + s\mathcal{L}\{f'(t)\}(s)$

$$= s^2F(s) - sf(0) - f'(0)$$