Math 4230
Wed 8/24
JTB 120

- roll
- syllabus
- → wiki
  history
  Plans
  Special.

HW for Friday Sept 2

1.1 1c, 2c, 4a, 6b, 7, 11, 14, 17a
1.2 1b, 2b, 3, 5, 8, 11, 14, 21
1.3 2a, 3b, 5b, 6b, 7a, 9, 10, 16ac, 22, 23, 24, 30ac

Def: the set of complex numbers, $\mathbb{C}$, is the real vector space $\mathbb{R}^2$ with the additional operation of complex multiplication:

$$(x_1, y_1) + (x_2, y_2) := (x_1 + x_2, y_1 + y_2) \qquad x_i, y_i, \alpha \in \mathbb{R}$$
$$\alpha(x, y) := (\alpha x, \alpha y)$$
$$(x_1, y_1)(x_2, y_2) := (x_1 x_2 - y_1 y_2, x_1 y_2 + y_1 x_2)$$

We usually write
$x + iy$ for (x, y)
and then the multiplication axiom reads $i^2 = -1$ and more generally

$$(x_1 + iy_1)(x_2 + iy_2) = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$$

$\mathbb{C}$ satisfies field axioms:

+
(i) $z + w = w + z$
(ii) $z + (w + s) = (z + w) + s$
(iii) $z + 0 = z$
(iv) $z + (-z) = 0$

mult
(i) $zw = wz$
(ii) $(zw)s = z(ws)$
(iii) $1z = z$
(iv) $\exists\ z^{-1}$ s.t. $zz^{-1} = 1$, $z \neq 0$

dist $z(w + s) = zw + zs$

} you get to check these in HW!

Notation: if $z = x + iy$ $\quad x, y \in \mathbb{R}$
then $\mathrm{Re}(z) := x$
$\mathrm{Im}(z) := y$

geometry of $\mathbb{C}$:

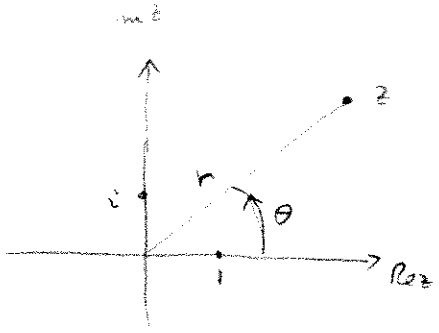
$z = x + iy$

$\bar{z} := x - iy$

$\overline{zw} = \bar{z}\bar{w}$ check!

$|z| := \sqrt{x^2+y^2} = \sqrt{z\bar{z}}$

"modulus" (also norm, abs val, mag.)



polar coords:

if $x = r\cos\theta$ $(r \geq 0)$

$y = r\sin\theta$

then $z = x+iy = r\cos\theta + ir\sin\theta$

$|z| = r$

$\arg(z) := \theta$ argument of z

if also $w = \rho\cos\phi + i\rho\sin\phi$

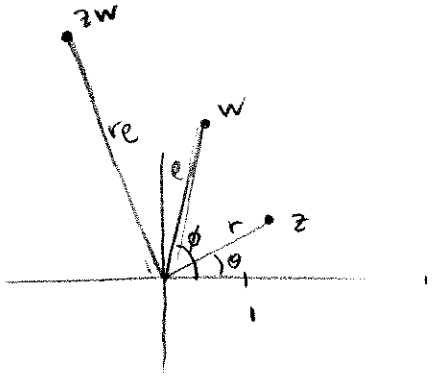
then

$$zw = (r\cos\theta + ir\sin\theta)(\rho\cos\phi + i\rho\sin\phi)$$
$$= r\rho\left[\cos\theta\cos\phi - \sin\theta\sin\phi + i(\cos\theta\sin\phi + \sin\theta\cos\phi)\right]$$
$$= r\rho\left[\cos(\theta+\phi) + i\sin(\theta+\phi)\right]$$

Geometric meaning of multiplication:

$|zw| = |z||w|$

$\arg(zw) = \arg(z) + \arg(w)$



<u>Cor</u> De Moivre's formula.

$n \in \mathbb{N}$

$z = r(\cos\theta + i\sin\theta)$

$z^n = r^n(\cos n\theta + i\sin n\theta)$

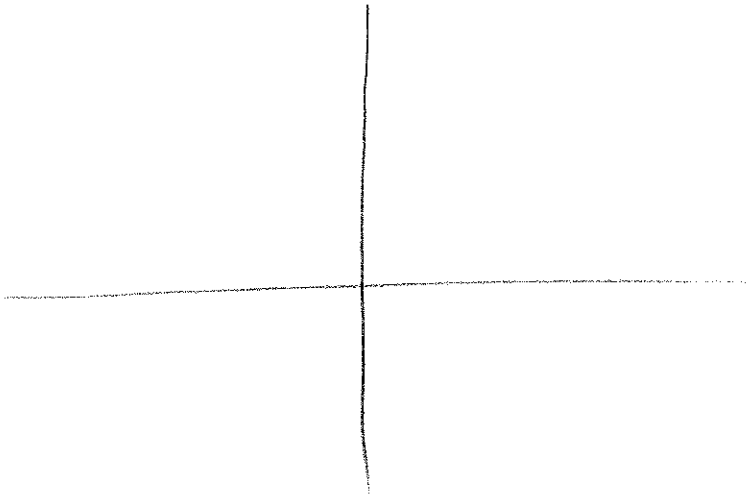
pf by induction via product formula.

Examples

Find $\frac{1}{3+4i}$ algebraically & geometrically

Solve $z^2 = 9i$ algebraically & geometrically

Find all sol's to $z^3 = 27$



Define $L: \mathbb{C} \to \mathbb{C}$ linear, by

$L(w) := (3+4i)w$

What is the matrix for L wrt the standard basis $\{1, i\}$?
What kind of matrix is this? (2270 language)