Pick up
- syllabus
- week 1 outline
- HW #1

Pick up & return
- sheet of paper ~ tell me briefly who you are & why you're interested in Math 2280.

Math 2280-002 Week 1 notes

We will not necessarily finish the material from a given day's notes on that day. We may also add or subtract some material as the week progresses, but these notes represent an in-depth outline of what we will cover. These notes are for sections 1.1-1.3, and part of 1.4.

Monday January 7

- Go over course information on syllabus and course homepage:

http://www.math.utah.edu/~korevaar/2280spring19

- Notice that there our first homework assignment is due next Wednesday, but that we will already have a quiz this Wednesday.

Then, let's begin!

Use notes as outline
work along in blank areas as we discuss topics

Syllabus for Math 2280-002 Differential Equations

Spring 2019

Instructor: Professor Nick Korevaar
email: korevaar@math.utah.edu
office: LCB 204, 801.581.7318
office hours: TBA
Lecture: MTWF 12:55-1:45 p.m. in LCB 219

Course websites

Lecture notes and homework assignments will be posted on our public home page.

http://www.math.utah.edu/~korevaar/2280spring19

There are blank spaces in the notes where we will work out examples and fill in details together. Research has shown that class attendance with active participation - including individual and collaborative problem solving, and writing notes by hand - are effective ways to learn class material for almost everyone. Passively watching a lecture without active participation is not usually very effective. Class notes will be posted on our public page, and I plan to bring weekly packets to class for you to use. Beyond what is outlined in the notes, there will be additional class discussion related to homework and other problems. Grades and exam material will be posted on our CANVAS course page; access via Campus Information Systems.

Textbook *Differential Equations and Boundary Value Problems, Computing and Modeling, 5th edition*, by Edwards and Penney. ISBN=978-0-321-79698-1.

Final Exam logistics: Tuesday April 30, 1:00 -3:00 p.m., in our classroom LCB 219. This is the University scheduled time and location.

Catalog description for Math 2280: Linear and nonlinear differential equations and systems of equations, with applications. Matrix exponential, fundamental solution matrix, phase-space and portraits, stability, initial- and boundary-value problems, introduction to partial differential equations. Requires familiarity with linear algebra. Includes theoretical and computer lab components.

matlab // wolfram alpha

Course overview: Math 2280 is an introduction to ordinary differential equations, and how they are used to model problems arising in engineering and science. It is the second semester of the year long sequence 2270-2280, which is an in-depth introduction to linear mathematics. Along with Calculus, the linear algebra that you learned in Math 2270 will provide a surprising amount of the conceptual and computational framework for our discussions in Math 2280. This will become more apparent as the course progresses.

Prerequisites: Linear Algebra, Math 2270, and its prerequisite courses of single and multi-variable calculus.

Students with disabilities:

The University of Utah seeks to provide equal access to its programs, services and activities for people with disabilities. If you will need accommodations in the class, reasonable prior notice needs to be given to the Center for Disability Services, 162 Olpin Union Building, 581-5020. CDS will work with you and the instructor to make arrangements for accommodations. All information in this course can be made available in alternative format with prior notification to the Center for Disability Services.

Grading

Math 2270-002 is graded on a curve. By this I mean that the final grading scale may end up lower than the usual 90/80/70% cut-offs. note: In order to receive a grade of at least C in the course you must earn a grade of at least C on the final exam. Typical final grade distributions in Math 2280 are roughly 40% each for As and Bs, and 20% with lower grades. Individual classes may vary. Details about the content of each assignment type, and how much they count towards your final grade are as follows:

- Homework (30%): There will be one homework assignment each week. Homework problems will be posted on our public page, and homework assignments will be due in class on Wednesdays. Homework assignments must be stapled. Unstapled assignments will not receive credit. I understand that sometimes homework cannot be completed on time due to circumstances beyond your control. To account for this, each student will be allowed to turn in two late homework assignments throughout the course of the semester. These assignments cannot be turned in more than one week late, and must be turned in on a Wednesday with the next homework assignment. You do not need to tell me the reason why your homework assignment is late. Homework will be a mixture of problems from the text and custom problems, and will vary from computational practice to modeling and more conceptual questions. There will be applications that require technology to complete. We will make use of Matlab for some of these applications.

- Quizzes (10%): At the end of most Wednesday classes, a short 1-2 problem quiz will be given, taking roughly 10 minutes to do. The quiz will cover relevant topics from the weeks lectures and homework. Your lowest two quiz scores will be dropped. There are no makeup quizzes. You will be allowed and encouraged to work together on these quizzes. starting this week

- Midterm exams (30%): Two class-length midterm exams will be given, On Friday February 15 and Friday March 29. I will schedule a room for review on the Thursday before each midterm, at our regular class time of 12:55-1:45 p.m. No midterm scores are dropped.

- Final exam (30%): A two-hour comprehensive exam will be given at the end of the semester. As with the midterms, a practice final will be posted. Please check the final exam time, which is the official University scheduled time. It is your responsibility to make yourself available for that time, so make any arrangements (e.g., with your employer) as early as possible.

Strategies for success:

- Attend and participate in class regularly.

- Read or at least scan the relevant text book sections and lecture note outlines *before* you attend class.

- Ask questions and become involved.

- Plan to do homework daily; try homework on the same day that the material is covered in lecture; do not wait until just before homework is due to begin serious work.

- Form study groups with other students.

(read on your own - we'll revisit this page at the end of the course to see how we did)

Learning objectives for Math 2280: The goal of Math 2280 is to master the basic tools and problem solving techniques important in differential equations, as well as to understand the larger conceptual framework that ties these ideas together. Specific goals include:

- Be able to model dynamical systems that arise in math, science and engineering, by using general principles to derive the governing differential equations or systems of differential equations. These principles include linearization, compartmental analysis, Newtons laws and conservation of energy.
- Learn solution techniques for first order separable and linear differential equations. Solve initial value problems in these cases, with applications to problems in science and engineering.
- Understand how to approximate solutions even when exact formulas do not exist. Visualize solution graphs and numerical approximations to initial value problems via slope fields. Understand phase diagram analysis for autonomous first order differential equations.
- Apply vector space concepts from linear algebra such as linear combinations, span, independence, basis and dimension, to understand the solution space to linear differential equations and linear systems of differential equations.
- Learn how to solve constant coefficient linear differential equations via superposition, particular solutions, and homogeneous solutions found via characteristic equation analysis. Apply these techniques to understand solutions to the basic unforced and forced mechanical and electrical oscillation problems.
- Learn how to use Laplace transform techniques to solve linear differential equations, with an emphasis on the initial value problems of mechanical systems, electrical circuits, and related problems.
- Understand the natural initial value problems for first order systems of differential equations, and how they encompass the natural initial value problems for higher order differential equations and general systems of differential equations.
- Be able to apply matrix algebra concepts related to eigenvalues, eigenvectors and matrix diagonalization, in order to find the general solution space to first and second order constant coefficient homogeneous linear systems of differential equations, especially those arising from compartmental analysis and mechanical systems.
- Learn how to work with matrix exponentials and other fundamental matrix solutions, as tools for understanding linear systems of differential equations with constant coefficients.
- Understand and be able to use linearization as a technique to understand the behavior of nonlinear dynamical systems near equilibrium solutions. Apply these techniques to autonomous systems of two first order differential equations, including interacting populations and systems arising from non-linear forced oscillation problems.
- Learn about Fourier series, and use them as an "infinite superposition" tool to study forced oscillation problems.
- Learn how to find solutions to Laplace's equation, the heat equation and the wave equation using separation of variables, together with Fourier series and superposition.
- Develop your abilities to model dynamical systems with differential equations, and to understand solutions analytically and with technology and software such as Matlab and internet-based tools as appropriate.

Week-by-Week Topics Plan

Topic schedule is subject to slight modifications as the course progresses, but exam dates are fixed.

Week 1: 1.1-1.4; differential equations and mathematical models; slope fields and initial value problems; separable differential equations.

Week 2: 1.5, 2.1-2.2 linear differential equations; applications of separable equations to population models; phase diagram analysis.

Week 3: 2.3-2.6; improved acceleration-velocity models; numerical solutions to DEs.

Week 4: 3.1-3.3; higher order linear differential equations; homogeneous and non-homogeneous problems.

Week 5: 3.4-3.6; applications to unforced mechanical vibrations; non-homogeneous linear differential equations and applications to forced mechanical vibrations.

Week 6: 3.6-3.7; applications and review. **Midterm exam 1 on Friday February 15** covering material from weeks 1-6.

Week 7: 7.1-7.4 Laplace transform approach to linear differential equations.

Week 8: 4.1, 4.3, 5.1-5.2; first order systems of differential equations, numerical methods, analytic solution of homogeneous first order systems with eigenvalue and eigenvector computations, input-output models.

Week 9: 5.2-5.4; mechanical vibrations and second order systems.

Week 10: 5.4-5.7; multiple eigenvalue solutions, matrix exponentials, and applications

Week 11: continued and review **Midterm exam 2 on Friday March 29**, covering weeks 7-11 material.

Week 12: 6.1-6.4; non-linear systems of first order differential equations with applications to ecological models and nonlinear mechanical systems.

Week 13: 9.1-9.4; Fourier series, with application to forced oscillations.

Week 14: 9.5-9.7; introduction to partial differential equations.

Week 15: continued, and course review.

Week 16: **Final exam Tuesday April 30**, 1:00-3:00 p.m. in our classroom LCB 219. This is the University scheduled time and location.

returning to Monday notes...

Section 1.1 Introduction to differential equations

Definition An n^{th} *order differential equation* (DE) is any equation expressed in terms of an uspecified function $y = y(x)$ and its derivatives, for which the highest derivative appearing in the equation is the n^{th} one, $y^{(n)}(x)$; i.e. any equation which after rearrangement can be written as

$$F\left(x, y(x), y'(x), y''(x), ... y^{(n)}(x)\right) = 0.$$

shorthand convention:

$$F\left(x, y, y', y'', y^{(n)}\right) = 0$$

Exercise 1: Which of the following are differential equations? For each DE determine the order.

a) For $y = y(x)$, $(y''(x))^2 + \sin(y(x)) = 0$. YES 2nd order

b) For $x = x(t)$, $x'(t) = 3\,x(t)(10 - x(t))$. YES [could rewrite as $x'(t) - 3x(t)(10 - x(t)) = 0$] 1st order

c) For $x = x(t)$, $x' = 3\,x\,(10 - x)$. YES (same as b)

d) For $z = z(r)$, $z'''(r) + 4\,z(r)$. NO not an equation! (equations have equal signs)

e) For $y = y(x)$, $y' = y^2$. YES. 1st order. (the "2" is a power, not a derivative).

f) For $y = y(t)$, $y' = y^{(2)}$ is a 2nd order DE

Related definitions:

- A specified function or functions $y(x)$ *solve(s)* the *differential equation*

$$F\left(x, y, y', y'', y^{(n)}\right) = 0$$

on some interval I of x-values (or is a *solution function* for the differential equation) means that $y(x)$ makes the differential equation a true identity for all x in I.

Chapters 1-2 are about first order differential equations, algebraic and graphical representations of their solutions, and applications. For first order differential equations

$$F(x, y, y') = 0$$

we can often use algebra to solve for y' in order to get what we call the **standard form** for the first order DE:

$$y' = f(x, y) \ .$$

• If we want our solution function to a first order DE to also satisfy $y\left(x_0\right) = y_0$, and if our DE is written in standard form, then we say that we are studying an *initial value problem (IVP):*

$$\text{IVP}\ \begin{cases} y' = f(x, y) \\ y\left(x_0\right) = y_0 \ . \end{cases} \leftarrow \text{initial condition}$$

If we can find a solution function $y(x)$ to the DE satisfying the *initial condition* $y\left(x_0\right) = y_0$, then we say that $y(x)$ *solves* the initial value problem.

Exercise 2: Consider the differential equation $\frac{dy}{dx} = y^2$ from (1e). i.e. $y'(x) = \left(y(x)\right)^2$

a) Show that functions $y(x) = \frac{1}{C - x}$ *solve* the DE (on any interval not containing the constant C). We'll see how we found these functions in part 2e, but we don't need that information to check whether or not they actually solve the differential equation.

b) Find the appropriate value of C to *solve the initial value problem*

$$\begin{cases} y' = y^2 \\ y(1) = 2 \ . \end{cases}$$

a) Does $y(x)$ make the DE a true identity?

$$y(x) = \frac{1}{C-x} = (C-x)^{-1}$$

then $y'(x) = +\frac{1}{(C-x)^2}$! LHS of DE

$$\frac{d}{dx}(C-x)^{-1} = -1(C-x)^{-2}(-1) = \frac{1}{(C-x)^2}$$

chain rule!

(alternate: quotient rule)

$$y(x)^2 = \left(\frac{1}{C-x}\right)^2 = \frac{1}{(C-x)^2} \quad \text{RHS of DE}$$

so for these functions the DE is a true identity (on any interval not containing $x = C$)

So, they are solutions

b) $y(x) = \dfrac{1}{C - x}$

want $y(1) = 2$: $\dfrac{1}{C-1} = 2$ solve for $C = 3/2$

$\left(C - 1 = \frac{1}{2} \right)$

$$\boxed{y(x) = \dfrac{1}{3/2 - x}}$$

2c) What is the largest interval on which your solution to 2b is defined as a differentiable function? Why?

$(-\infty, 3/2)$

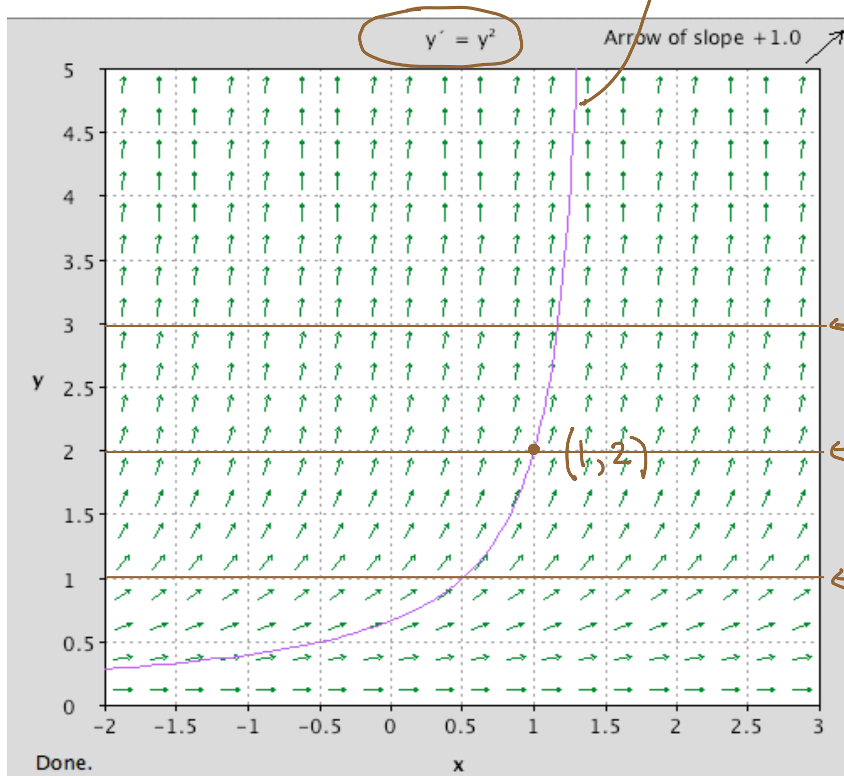
i.e. $-\infty < x < 3/2$

2d) Do you expect that there are any other solutions to the IVP in 2b (on the same interval)? Hint: The graph of the IVP solution function we found is superimposed onto a "slope field" below, where the line segment slopes at points (x, y) have values y^2 (because solution graphs to our differential equation will have those slopes, according to the differential equation). This might give you some intuition about whether you expect more than one solution to the IVP.

We expect only one possible graph that is tangent to the slope field and that goes through the initial point $(1,2)$.

$y' = y^2$ $\qquad$ $y = \dfrac{1}{3/2 - x}$

at points (x,y) slopes of green segments are given by "y^2" for graph of any soln to this DE, the graph will be tang. to slope field



all slopes are 9.

all slopes = 4

all slopes = 1