Math 2280-002

Week 2 concepts and homework, due Wednesday January 23 at the start of class

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Wednesday quiz will be drawn from all of these concepts and from these or related problems.

1.5: **33**, **34**, **36**

2.1: 1, **3**, **6**, **10**, **12**, **23**, 33 (In 23, notice this is just a logistic DE, so you may use the general solution formula.)

2.2: 5, 7, 9, 11

w2.1) (section 1.4 Newton's law of cooling modeling) As part of the summer job at a restaurant, you learned to cook up a big pot of soup late at night, just before closing time, so that there would be plenty of soup to feed customers the next day. However the soup was too hot to be put directly into the fridge when it was ready. (The soup had just boiled at $100^o\ C$, and the fridge was not powerful enough to accomodate a big pot of soup if it was any warmer than $20^o\ C$.) Suppose that by cooling the pot in a sink full of cold water, (kept running, so that its temperature was roughly constant at $5^o\ C$) and stirring occasionally, you could bring the temperature of the soup to $60^o\ C$ in 10 minutes. How long before closing time should the soup be ready so that you could put it in the fridge and leave on time?

w2.2) (section 1.5 modeling) A 25-year-old woman accepts an actuarial position with a starting salary of *\$70,000* per year. her salary $S(t)$ increases exponentially at a continuous rate of *5 %* per year, so that $S(t) = 70\, e^{.05\, t}$ thousand dollars per year, after t years. To save for retirement she deposits *10 %* of her salary continuously into a retirement account, which accumulates interest at an annual rate of *4 %* per year. Let $A(t)$ be the amount in the retirement account after t years, with $A(0) = 0$ thousands of dollars at the time she begins her new job.

a) Estimate the change ΔA in terms of Δt to derive the differential equation for $A(t)$.

b) Compute the amount of money she will have in her retirement account if she retires at age 67.

w2.3 This is a continuation of 2.1.23. Create a dfield plot for the logistic differential equation in this problem, say with $0 \le t \le 2$ and $0 \le x \le 150$, and verify that your answer to 2.1.23b is consistent with the dfield prediction, by adding the IVP solution to the dfield plot and verifying with your cursor that the *t*-value at which the solution graph passes through the horizontal line $x = 100$ agrees with your answer to 23b.

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Note added 1/22: *problem w2.4 and the unbolded section 2.2 material above is postponed until the next homework assignment.*

w2.4) *Consider the differential equation*

$$\frac{dx}{dt} = x^4 - 9x^2 .$$

a) *Find the equilibria; draw the phase portrait;*

b) *classify the equilbria as stable, asymptotically stable, or unstable (possibly one-sided stable);*

c) *use dfield to sketch the slope field and representative solution graphs, including the graphs of the equilibrium solutions, to verify your phase portrait analysis. Include this plot in your homework.*