Math 2280-002
Week 14 concepts and homework
9.4-9.6
Due Tuesday April 23
(accepted until 5:00 p.m. April 24, slide it under my office door LCB 204 if I'm not there)

9.4 Forced oscillation with periodic forcing functions
1, 13

9.5 Heat equation
1, 2, 11, 13. For 13 use the table of diffusivity constants:

Material	k (cm²/s)
Silver	1.70
Copper	1.15
Aluminum	0.85
Iron	0.15
Concrete	0.005

FIGURE 9.5.3. Some thermal diffusivity constants.

9.6 Wave equation
1, **5**, **10**, **13**, **14**, 15, 16.

w14.1a Let $f(t)$ be the 6π - periodic forcing function with

$$f(t) = \begin{cases} 1 & 0 < t < \pi \\ 0 & \pi < t < 6\pi \end{cases}$$

Find the Fourier series for $f(t)$. Hint: Your a_n and b_n will involve $\sin\left(\frac{n\pi}{3}\right)$ and $\cos\left(\frac{n\pi}{3}\right)$.

b Find the general solution to

$$x''(t) + x(t) = f(t),$$

using the homogeneous solution and infinite superposition of particular solutions, as we've done in class. You may use the table from class (which is also appended to this homework assignment).

c Identify the term in your solution which shows the resonance. (See example 5 in the April 5 class notes, for a related discussion, as well as class discussion April 15.)

w14.2 Now consider the slightly damped forced oscillation problem

$$x''(t) + .2\,x'(t) + x(t) = f(t)$$

with the same forcing function as in **w14.1**. Write down the formal series for the steady periodic solution in this case, and identify the term which is causing the practical resonance. (See text and class discussion for related problem on April 16.)

Particular solutions from Chapter 3:

$$x''(t) + \omega_0^2 x(t) = A\,sin(\omega t)$$

$$x_P(t) = \frac{A}{\omega_0^2 - \omega^2}\sin(\omega t) \quad \text{when } \omega \neq \omega_0$$

$$x_P(t) = -\frac{t}{2\,\omega_0} A\cos(\omega_0 t) \quad \text{when } \omega = \omega_0$$

....

$$x''(t) + \omega_0^2 x(t) = A\cos(\omega t)$$

$$x_P(t) = \frac{A}{\omega_0^2 - \omega^2}\cos(\omega t) \quad \text{when } \omega \neq \omega_0$$

$$x_P(t) = \frac{t}{2\,\omega_0} A\sin(\omega_0 t) \quad \text{when } \omega = \omega_0$$

....

$$x'' + c\,x' + \omega_0^2 x = A\cos(\omega t) \qquad c > 0$$

$$x_P(t) = x_{sp}(t) = C\cos(\omega t - \alpha)$$

with

$$C = \frac{A}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + c^2\omega^2}}\ .$$

$$\cos(\alpha) = \frac{\omega_0^2 - \omega^2}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + c^2\omega^2}}$$

$$\sin(\alpha) = \frac{c\,\omega}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + c^2\omega^2}}\ .$$

....

$$x'' + c\,x' + \omega_0^2 x = A\sin(\omega t) \qquad c > 0$$

$$x_P(t) = x_{sp}(t) = C\sin(\omega t - \alpha)$$

with

$$C = \frac{A}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + c^2\omega^2}}\ .$$

$$\cos(\alpha) = \frac{\omega^2 - \omega_0^2}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + c^2\omega^2}}$$

$$\sin(\alpha) = \frac{c\,\omega}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + c^2\omega^2}}$$