Math 2280-002
Week 13 concepts and homework
9.1-9.3
Due Wednesday April 17

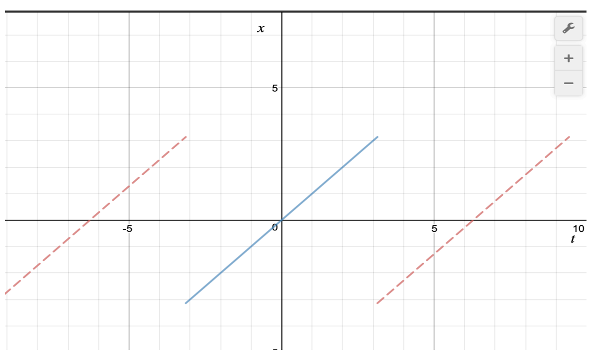
*9.1 Fourier series for $2\pi$ - periodic functions*
1, **2**, **3**, **9**, **10**, **30** (30 is important because it shows we can use any interval of length $2\pi$ to compute the Fourier coefficents of a $2\pi -$ periodic function; or any interval of length $2L$ for the Fourier coefficients of a $2L$ periodic function.)

*9.2 General Fourier series for $2L$ - periodic functions*
**2**, **9**

*9.3 sine and cosine series for odd and even extensions of functions defined on $[0, L]$*
**1**, **5**, **10**

**w13.1a** Find the Fourier series for the "sawtooth" function $saw(t)$, which is defined as the the $2\pi$ - periodic extension of

$$f(t) = t, \quad -\pi < t < \pi$$




Your answer should be

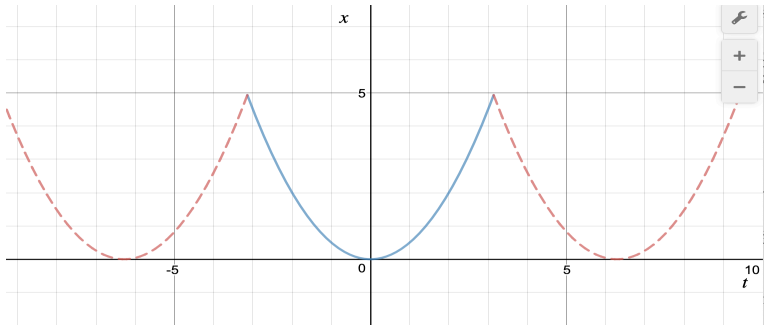
$$saw(t) \sim 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(n\,t).$$

**b** Create a plot at Desmos (or on other software) of the truncated Fourier series, containing the first 5 non-zero terms. Hand in a screen shot of your result.

**w13.2a** Use the fact that you can integrate Fourier series term by term, and the result of **w13.1** to find the Fourier series for the $2\pi$ - periodic extension of

$$f(t) = \frac{t^2}{2}, \quad -\pi < t < \pi .$$

Hint: Use the usual formula for $\frac{a_0}{2}$ to find the constant of integration.




**b** Since this function is piecewise differentiable and continuous, the Fourier series converges at every $t-$ value, to the original function. Evaluate the Fourier series at $t = 0$ and at $t = \pi$ to get two mysterious equalities relating $\pi^2$ to two different infinite series.

**w13.3** Notice that the square wave $sq(t)$ from class (and the text) is related to the $f(t)$ in **9.2.2**. In fact,

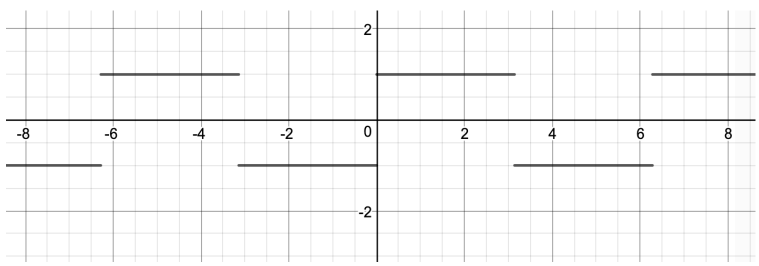
$$f(t) = \frac{1}{2} + \frac{1}{2} square\left(\frac{\pi}{5}t\right).$$

Use this connection, and the fact that $sq(t)$ has Fourier series

$$sq(t) \sim \frac{4}{\pi}\sum_{n\ odd}\frac{1}{n}\sin(n\,t)$$

to check your work in 9.2.2. (Rescaling Fourier series in the domain and range, to get related Fourier series, can be very useful.)

The graph of $sq(t)$:




The graph of the period 10 extension of

$$f(t) = \begin{cases} 0 & -5 < t < 0 \\ 1 & 0 < t < 5 \end{cases}$$