Math 2280-002

Week 1 concepts and homework, due Wednesday January 16 (at the start of class).

All of the indicated problems are good for seeing if you can work with the underlying concepts. Only the underlined problems are to be handed in; the others might be useful to look at and/or try if you want more practice. Please arrange your homework solutions so that the text problems come before the custom problems, to help make our grader's life easier.

1.1: 1, 4, 5, **6**, 9, **15**, 19, **27**, **29**, **30**, 32, 33, **34**.

1.2: 1, **2**, 5, **6**, **7**, **9**, 10, 13, 15, **16**, **18**, **21**, **24** , **26** , 31, 32, 33.

1.3: **2**, 3, 5, **6**, 10, **11**, 12, 13, 14, 17, 18.

1.4: 2, 3, **4**, 9, **12**,13,19, **20**, 21, **36**, 41, 42, 45, **46**, 49, 50, **54** , **66**.

1.5: **1**, **7**, 8, **13**, **14**, **24**, **29**

Custom problems:

week 1.1) Solve the following initial value problems from section 1.2 as a way to review important integration techniques from Calculus: substitution and integration by parts.

a) $\frac{dy}{dx} = 2\sin\left(\frac{x}{3}\right)$, $y(0) = 2$.

b) $y'(x) = 3\,x\,e^{-x}$, $y(0) = 0$.

c) $\frac{dy}{dx} = \frac{2\,x}{\sqrt{x^2 + 9}}$, $y(0) = 3$.

w1.2 Suppose an object moves along a real number line, with position function $x(t)$ at time t and with constant negative acceleration,

$$x''(t) = -a, \;\; a > 0.$$

w1.2a) Let $x(0) = 0$, $x'(0) = v_0 > 0$. Show that the maximum x-value is given by

$$x_{max} = \frac{1}{2}\,\frac{v_0^2}{a}.$$

Hint: Find and use the formulas for velocity $v(t) = x'(t)$ and position $x(t)$.

w1.2b) Car accident reconstruction. A driver skids 210 ft. after applying his brakes. He claims to the investigating officers that he was going 25 miles per hour before trying to stop. A police test of his vehicle shows that if the brakes are applied to force a skid at an initial speed of 25 $\frac{mi}{h}$ then the auto skids only 45 ft . Assuming that the car is decelerating at a constant rate while skidding, about how fast was the driver really going? Hint: One way to do this problem is to use the formula you derived for x_{max} in part **a**. You have two x_{max} values, one initial velocity you know, and one you don't.

w1.3

Consider the differential equation from 1.3.6:

$$y'(x) = x - y + 1.$$

a) Show that the functions $y(x) = x + C\,e^{-x}$ solve this differential equation.

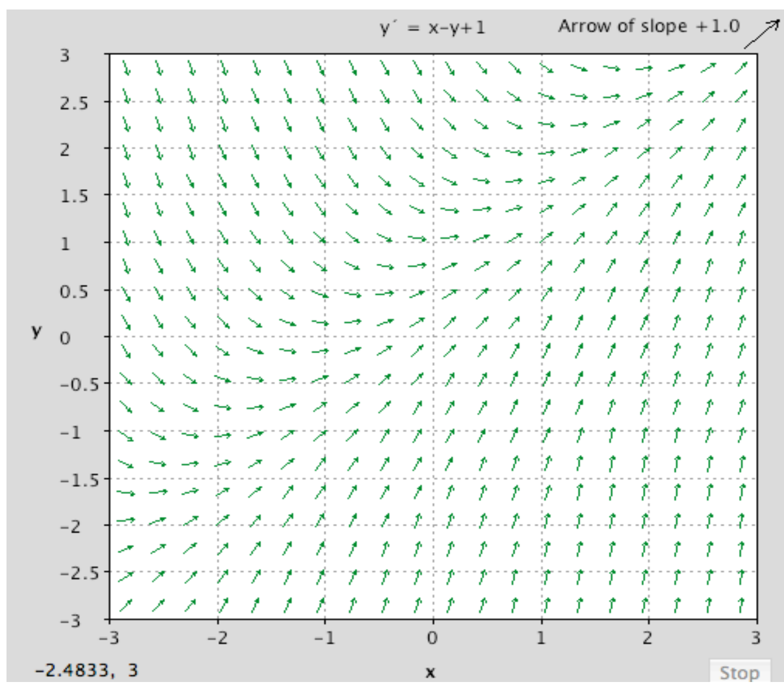
b) Find the value of C in the general solution above, so that $y(x)$ solves the initial value problem

$$y'(x) = x - y + 1$$
$$y(-1) = 2.$$

Identify the graph of this solution on the slope field below.

c) Add the diagonal asymptote for this graph, and write its equation. Notice that the diagonal asymptote is itself the graph of one of the solution functions. The slope field picture has been created using the JAVA applet "dfield", at the URL

http://math.rice.edu/~dfield/dfpp.html



w1.4) In homework problem 1.1.29 you showed that if a function $y = y(x)$ has the property that every straight line normal to the graph $y = y(x)$ passes through the point $(0, 1)$, then $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = -\frac{x}{y - 1}.$$

a) Use the existence-uniqueness theorem to verify that the initial value problem for this differential equation, with $y(0) = -2$, has a unique solution on some interval containing $x_0 = 0$.

b) Use dfield to create a slope field for this differential equation inside the rectangle $-4 < x < 4, -4 < y < 4$. Have dfield find representative solution graphs. What do you notice about their shapes? Does this make sense based on the original geometric description that led to this differential equation? Explain. Hint: You can download dfield from the website

http://math.rice.edu/~dfield/dfpp.html

c) Solve the initial value problem in part **a**, to explicitly find $y(x)$. What is the largest x-interval on which $y(x)$ can be defined as a differentiable function? Explain.

w1.5) Consider the differential equation we study in class on Wednesday and Friday of week 1, but this time for a different initial value problem:

$$\frac{dy}{dx} = y^{\left(\frac{2}{3}\right)}$$
$$y(0) = 1.$$

a) Use the existence-uniquess theorem to show this IVP has a unique solution on some interval containing $x_0 = 0$.

b) Use separation of variables to find this solution.

c) What is the largest interval on which the solution you found in **b** is the unique solution to the IVP? Explain.