Math 2280-001
Lab 5
These are selected problems from the homework due Friday

w5.5) Consider the three functions

$$y_1(x) = \cos(2x), \quad y_2(x) = \sin(2x), y_3(x) = \sin\left(2x - \frac{\pi}{6}\right).$$

w5.5a) Show that all three functions solve the differential equation

$$y'' + 4y = 0 .$$

w.5b) The differential equation above is a second order linear homogeneous DE, so the solution space is 2-dimensional. Thus the three functions y_1, y_2, y_3 above must be linearly dependent. Find a linear dependency. (Hint: use a trigonometry addition angle formula.)

w5.5c) Explicitly verify that every initial value problem

$$y'' + 4y = 0$$
$$y(0) = b_1$$
$$y'(0) = b_2$$

has a solution of the form $y(x) = c_1\cos(2x) + c_2\sin(2x)$, and that c_1, c_2 are uniquely determined by b_1, b_2 . (Thus $\cos(2x), \sin(2x)$ are a basis for the solution space of $y'' + 4y = 0$: every solution $y(x)$ has initial values that can be matched with a linear combination of y_1, y_2 , but once the initial values match the solutions must agree by the uniqueness theorem, so y_1, y_2 span the solution space; y_1, y_2 are linearly independent because if $c_1\cos(2x) + c_2\sin(2x) = y(x) \equiv 0$ then $y(0) = y'(0) = 0$ so also $c_1 = c_2 = 0$.)

w5.5d) Find by inspection, particular solutions $y(x)$ to the two non-homogeneous differential equations

$$y'' + 4y = 28, \qquad y'' + 4y = -16x$$

Hint: one of them could be a constant, the other could be a multiple of x.

w5.5e) Use superposition (linearity) and your work from **c,d** to find the general solution to the non-homogeneous differential equation

$$y'' + 4y = 28 - 16x.$$

w5.5f) Solve the initial value problem, using your work above:

$$y'' + 4y = 28 - 16x$$
$$y(0) = 0$$
$$y'(0) = 0.$$

w5.4) Consider the 3^{rd} order homogeneous linear differential equation for $y(x)$

$$y'''(x) = 0$$

and let W be the solution space.

w5.4a) Use successive antidifferentiation to solve this differential equation. Interpret your results using vector space concepts to show that the functions $y_0(x) = 1, y_1(x) = x, y_2(x) = x^2$ are a basis for W. Thus the dimension of W is 3.

w5.4b) Show that the functions $z_0(x) = 1, z_1(x) = x - 2, z_2(x) = \frac{1}{2}(x-2)^2$ are also a basis for W.

Hint: If you verify that they solve the differential equation and that they're linearly independent, they will automatically span the 3-dimensional solution space and therefore be a basis.

w5.4c) Use a linear combination of the solution basis from part b, in order to solve the initial value problem below. Notice how this basis is adapted to initial value problems at $x_0 = 2$, whereas for an IVP at $x_0 = 0$ the basis in a would have been easier to use.

$$y'''(x) = 0$$
$$y(2) = 7$$
$$y'(2) = -13$$
$$y''(2) = 5.$$