Math 2280-001
Lab 4
These are selected problems from the homework due Friday

week 4.1 Consider a bioreactor used by a yogurt factory to grow the bacteria needed to make yogurt. The growth of the bacteria is governed by the logistic equation

$$\frac{dP}{dt} = k \cdot P(M - P)$$

where P is the population in millions and t is the time in days. Recall that M is the carrying capacity of the reactor, and k is a constant that depends on the growth rate.

a) Through observation it is found that after a long time the population in the reactor stabilizes at 50 million bacteria, and that when the population of the reactor is 20 million bacteria the population increases at a rate of 12 million per day. From this, find k and M in the governing equation.

b) If the colony starts with a population of 10 million bacteria, how long will it take for the population to reach 80 % of carrying capacity?

c) Suppose the factory harvests the bacteria from the reactor once a week. The harvesting process takes a day, during which the reactor is not operational, leaving 6 days per week for the bacteria to grow in the reactor. The factory wants to maximize the amount of bacteria grown during these 6 days. To achieve this, $P'(t)$ should be at its maximum 3 days after harvesting. What initial population (after harvesting) gives the most growth over the 6-day period? What is the population change during this time?

d) Suppose the reactor is modified to allow for continual harvesting without shutting down the reactor. Let h be the rate at which the bacteria are harvested, in millions per day. Write down the new differential equation governing the bacteria population. What is the maximum rate of harvesting h that will not cause the population of bacteria to go extinct? (Harvesting at less than this rate will ensure that there is always a stable equilibrium point where P is positive.)

Hints: for parts b,c it might be helpful to recall and use the solution to the logistic DE IVP:

$$P(t) = \frac{MP_0}{(M - P_0)e^{-Mkt} + P_0}$$

<u>2.3.10</u>) A woman bails out of an airplane at an altitude of 10,000 ft, falls freely for 20 s, then opens her parachute. How long will it take her to reach the ground? Assume linear air resistance $\rho\, v\, \frac{ft}{s^2}$, taking $\rho = 0.15$ without the parachute and $\rho = 1.5$ with the parchute. (*Suggestion:* First determine her height above the ground and velocity, when the parachute opens.)

2.3.12) It is proposed to dispose of nuclear wastes - in drums with weight $W = 640\ lb$ and volume $8\ ft^3$ - by dropping them into the ocean ($v_0 = 0$). The force equation for a drum falling through the water is

$$m\,v'(t) = -W + B + F_R,$$

where the buoyant force B is equal to the weight (at $62.5\ \frac{lb}{ft^3}$) of the volume of water displaced by the drum (Archimedes' principle) and F_R is the force of the water resistance, found empirically to be $1\ lb$ for each $\frac{ft}{s}$ of the velocity of the drum. If the drums are likely to burst upon an impact of more than $75\ \frac{ft}{s}$, what is the maximum depth to which they can be dropped in the ocean without likelihood of bursting? (Hint: since the force of gravity, i.e. the weight of the drum, is given by $F = mg$ the mass m in slugs is obtained from the weight by dividing by $g = 32\ \frac{ft}{s^2}$.)