Math 2280-001
Lab 3
These are selected problems from the homework due Friday

1.5.38) Consider a cascade of two tanks shown below, with $V_1 = 100$ (gallons) and $V_2 = 200$ (gal) the volumes of brine in the two tanks. Each tank initially contains 50 lb of salt. The three flow rates indicated in the figure are eqch 5 $\frac{gal}{\text{min}}$, with pure water flowing into tank 1.

a) Find the amount $x(t)$ of salt in tank 1 at time t. (Hint: the differential equation for $x(t)$ turns out to be an exponential decay DE, so you can just recall the solution from section 1.4.)

b) Suppose that $y(t)$ is the amount of salt in tank 2 at time t. Show that the DE for $y(t)$ is

$$y'(t) = .05\,x - .025\,y.$$

Then solve the initial value problem for $y(t)$.

c) Find the maximum amount of salt ever in tank 2.

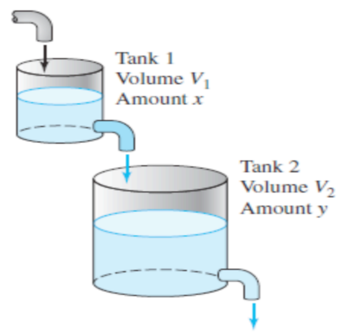

FIGURE 1.5.5. A cascade of two tanks.

w3.1) (lab) A 25-year-old woman accepts an actuarial position with a starting salary of $70,000 per year. her salary $S(t)$ increase exponentially at a continuous rate of 5 % per year, so that $S(t) = 70\, e^{.05\, t}$ thousand dollars per year, after t years. To save for retirement she deposits 10 % of her salary continuously into a retirement account, which accumulates interest at an annual rate of 4 % per year. Let $A(t)$ be the amount in the retirement account after t years, with $A(0) = 0$ thousands of dollars at the time she begins her new job.

a) Estimate the change ΔA in terms of Δt to derive the differential equation for $A(t)$.

b) Compute the amount of money she will have in her retirement account if she retires at age 67.

w3.3) (lab) In problem **2.1.23**, use dfield to plot the differential equation slope field for $0 \leq t \leq 3$ and $-50 \leq x \leq 250$. Use the dfield option which lets you specify initial values in order to add the graph of the solution to the IVP in 23, along with the graphs of the two equilibrium solutions. Notice that as you move your mouse over the dfield plot, its location is tracked by the dfield applet. Use your mouse to find the time when the solution to the IVP in 23 satisfies $x(t) = 100$, and record this information. Print out a copy of your plot; add the coordinates of this intersection point at which $x = 100$. Is the $t-$ coordinate of this point consistent with your work in 23b?

For your reference, here is 2.1.23, which you want to do in lab if you haven't done it already.
2.1.23) As the salt KNO_3 dissolves in methanol, the number $x(t)$ of grams of the salt in solution after t seconds satisifes the differential equation

$$x'(t) = 0.8\,x - .004\,x^2.$$

a) What is the maximum amount of the salt that will ever dissolve in the methanol?
b) If $x(0) = 50$, how long will it take for an additional 50 g to dissolve?