Math 2280-001
Lab 2
These are selected problems from the homework due Friday

w2.1 Suppose an object moves in linear motion, with position function $x(t)$ at time t and with constant negative acceleration,

$$x''(t) = -a, \ a > 0.$$

w2.1a) Let $x(0) = 0$, $x'(0) = v_0 > 0$. Show that the maximum x-value is given by

$$x_{max} = \frac{1}{2}\frac{v_0^2}{a}.$$

Hint: Find and use the formulas for velocity $v(t) = x'(t)$ and position $x(t)$.
w2.1b) Car accident reconstruction. A driver skids 210 ft. after applying his brakes. He claims to the investigating officers that he was going 25 miles per hour before trying to stop. A police test of his vehicle shows that if the brakes are applied to force a skid at an initial speed of 25 $\frac{mi}{h}$ then the auto skids only 45 ft. Assuming that the car is decelerating at a constant rate while skidding, about how fast was the driver really going? Hint: One way to do this problem is to use the formula you derived for x_{max} in part **a**. You have two x_{max} values, one initial velocity you know, and one you don't.

w2.2) In problem in 1.3.11 you (will) show that the IVP

$$y'(x) = 2x^2y^2$$
$$y(1) = -1.$$

has a unique solution on some interval containing $x_0 = 1$. In that problem you are supposed to use the existence uniqueness theorem to find a coordinate rectangle R containing the initial point $(1, -1)$ in the interior so that the slope function $f(x, y) = 2x^2y^2$ is continuous in R (this guarantees that the IVP has at least one solution), and so that $\frac{\partial}{\partial y} f(x, y)$ is also continuous in R (this guarantees that the solution is unique for as long as its graph stays in R. If you haven't completed 1.3.11 yet you may want to do it here:
1.3.11)

a) The differential equation in this problem is separable, so you can actually find the solution to the initial value problem above. Do so.

b) What is the largest interval on which the solution $y(x)$ to **b** is defined as a differentiable function? Explain. Hint: The graph will have a vertical asymptote.

w2.3) In homework problem 1.1.29 last week you showed that if a function $y = y(x)$ has the property that every straight line normal to the graph $y = y(x)$ passes through the point $(0, 1)$, then $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = -\frac{x}{y-1} .$$

a) Use the existence-uniqueness theorem to verify that the initial value problem for this differential equation, with $y(0) = -2$, has a unique solution on some interval containing $x_0 = 0$. Hint: what line must your rectangle R avoid?

b) Use dfield to create a slope field for this differential equation inside the rectangle $-4 < x < 4, -4 < y < 4$. Have dfield find representative solution graphs. What do you notice about their shapes? Does this make sense based on the original geometric description that led to this differential equation? Explain. Hint: You can download dfield from the website

http://math.rice.edu/~dfield/dfpp.html

c) Solve the initial value problem in part **a**, to explicitly find $y(x)$ (after you find it implicitly using the method of separable differential equations). What is the largest x-interval on which $y(x)$ can be defined as a differentiable function? Explain.

w2.4) (lab) Consider the differential equation we studied in class on Friday January 13, but this time for a different initial value problem:

$$\frac{dy}{dx} = y^{\left(\frac{2}{3}\right)}$$

$$y(0) = 1.$$

a) Use the existence-uniquess theorem to show this IVP has a unique solution on some interval containing $x_0 = 0$.

b) Use separation of variables to find this solution.

c) What is the largest interval on which the solution you found in **b** is the unique solution to the IVP? Explain.

Here's what's going on (stated in 1.3 page 24 of text; partly proven in Appendix A.)

Existence - uniqueness theorem for the initial value problem

Consider the IVP

$$\frac{dy}{dx} = f(x, y)$$
$$y(a) = b$$

- Let the point (a, b) be interior to a coordinate rectangle $\mathcal{R}: a_1 \leq x \leq a_2, b_1 \leq y \leq b_2$ in the $x-y$ plane.
- Existence: If $f(x, y)$ is continuous in $\mathcal{R}$ (i.e. if two points in $\mathcal{R}$ are close enough, then the values of f at those two points are as close as we want). Then there exists a solution to the IVP, defined on some subinterval $J \subseteq [a_1, a_2]$.
- Uniqueness: If the partial derivative function $\frac{\partial}{\partial y} f(x, y)$ is also continuous in $\mathcal{R}$, then for any subinterval $a \in J_0 \subseteq J$ of x values for which the graph $y = y(x)$ lies in the rectangle, the solution is unique!

See figure below. The intuition for existence is that if the slope field $f(x, y)$ is continuous, one can follow it from the initial point to reconstruct the graph. The condition on the y-partial derivative of $f(x, y)$ turns out to prevent multiple graphs from being able to peel off.