Math 2280-001

Week 2 concepts and homework, due January 20.

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems. The tentative Tuesday lab problems are indicated by (lab),

problems from section 1.2:

w2.1 (lab) Suppose an object moves in linear motion, with position function $x(t)$ at time t and with constant negative acceleration,

$$x''(t) = -a, \; a > 0.$$

w2.1a) Let $x(0) = 0$, $x'(0) = v_0 > 0$. Show that the maximum x-value is given by

$$x_{max} = \frac{1}{2}\frac{v_0^2}{a}.$$

Hint: Find and use the formulas for velocity $v(t) = x'(t)$ and position $x(t)$.

w2.1b) Car accident reconstruction. A driver skids 210 ft. after applying his brakes. He claims to the investigating officers that he was going 25 miles per hour before trying to stop. A police test of his vehicle shows that if the brakes are applied to force a skid at an initial speed of 25 $\frac{mi}{h}$ then the auto skids only 45 ft. Assuming that the car is decelerating at a constant rate while skidding, about how fast was the driver really going? Hint: One way to do this problem is to use the formula you derived for x_{max} in part **a**. You have two x_{max} values, one initial velocity you know, and one you don't.

1.3 Slope fields and solution curves: understand how the graph of the solution to a first order DE IVP is related to the underlying slope field; the existence-uniqueness theorem for solutions to IVPs.

1.3: ***11***, *12, 13, 14, 17, 18.*

w2.2) (lab) In problem in 1.3.11 above you show that the IVP

$$y'(x) = 2x^2y^2$$
$$y(1) = -1.$$

has a unique solution on some interval containing $x_0 = 1$.

a) The differential equation in this problem is separable, so you can actually find the solution to the initial value problem above. Do so.

b) What is the largest interval on which the solution $y(x)$ to **b** is defined as a differentiable function? Explain. Hint: The graph will have a vertical asymptote.

w2.3) (lab) In homework problem 1.1.29 last week you showed that if a function $y = y(x)$ has the property that every straight line normal to the graph $y = y(x)$ passes through the point $(0, 1)$, then $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = -\frac{x}{y-1}.$$

a) Use the existence-uniqueness theorem to verify that the initial value problem for this differential equation, with $y(0) = -2$, has a unique solution on some interval containing $x_0 = 0$.

b) Use dfield to create a slope field for this differential equation inside the rectangle $-4 < x < 4, -4 < y < 4$. Have dfield find representative solution graphs. What do you notice about their shapes? Does this make sense based on the original geometric description that led to this differential equation? Explain. Hint: You can download dfield from the website

http://math.rice.edu/~dfield/dfpp.html

c) Solve the initial value problem in part **a**, to explicitly find $y(x)$. What is the largest x-interval on which $y(x)$ can be defined as a differentiable function? Explain.

w2.4) (lab) Consider the differential equation we studied in class on Friday January 13, but this time for a different initial value problem:

$$\frac{dy}{dx} = y^{\left(\frac{2}{3}\right)}$$

$$y(0) = 1.$$

a) Use the existence-uniquess theorem to show this IVP has a unique solution on some interval containing $x_0 = 0$.

b) Use separation of variables to find this solution.

c) What is the largest interval on which the solution you found in **b** is the unique solution to the IVP? Explain.

1.4: *2,3,**4**,9,**12**,13,19,**20**,21: solving DE's and IVP's for separable differential equations*

__36__, 41, 42, 45, __46__, 49, 50, __54__ (lab) modeling and solving problems with first order separable DEs.

w2.5 As part of the summer job at a restaurant, you learned to cook up a big pot of soup late at night, just before closing time, so that there would be plenty of soup to feed customers the next day. However the soup was too hot to be put directly into the fridge when it was ready. (The soup had just boiled at 100^o *C*, and the fridge was not powerful enough to accomodate a big pot of soup if it was any warmer than 20^o *C*.) Suppose that by cooling the pot in a sink full of cold water, (kept running, so that its temperature was roughly constant at 5^o *C*) and stirring occasionally, you could bring the temperature of the soup to 60^o *C* in 10 minutes. How long before closing time should the soup be ready so that you could put it in the fridge and leave on time?