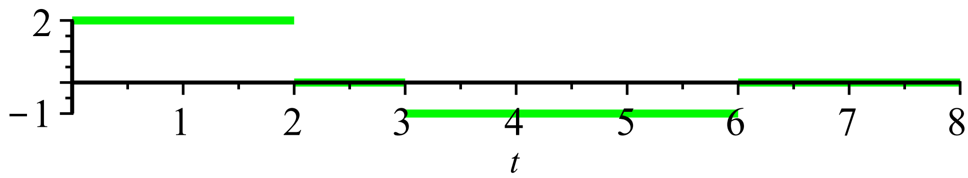
Math 2280-001
Week 13 concepts and homework, due April 18 at the start of lab

7.5 using the unit step function to turn forcing on and off (discussed in class on Monday April 10).

w13.1 (lab) Consider the piecewise constant function $f(t)$ from your previous homework problem **w12.4**:



a) Express $f(t)$ as a linear combination of translated unit step functions,
b) Compute the Laplace transform of $f(t)$ using the table entry.

$u(t-a)$	$\frac{e^{-as}}{s}$

7.5.31 (lab) Write the forcing function as a linear combination of unit step functions, in order to solve the IVP

$$x''(t) + 4x(t) = f(t)$$
$$x(0) = 0$$
$$x'(0) = 0$$

Here the forcing function is given by

$$f(t) = \begin{cases} 1 & 0 \le t < \pi \\ 0 & t \ge \pi \end{cases}.$$

You will make use of the table entry below, to find the inverse Laplace transform of $X(s)$

$u(t-a)f(t-a)$	$e^{-as}F(s)$

7.4 convolutions (discussed in class on Wednesday April 12).
2, 13, **36**, **37**. (Note, 36, 37 are very quick applications of the convolution table entry.)

w13.2a) (lab) Compute the convolution $f*g(t)$ for $f(t) = t$ and $g(t) = t^2$ and use the Laplace transform table to show that $\mathcal{L}\{f*g(t)\}(s) = F(s)G(s)$ in this case. Be clever about which order you write the f, g terms in the convolution integral, as one way is easier than the other.
b) Repeat part a for the example $f(t) = t,\ g(t) = \sin(k\,t)$. The convolution table entry is

$f*g(t) := \int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$	convolution integrals to invert Laplace transform products

EP 7.6 Engineering applications: Duhamel's Principle and delta function forcing.
EP 7.6 1, **2**

w13.3) Redo **7.5.31** using the convolution table entry to find the solution $x(t)$ to the initial value problem:

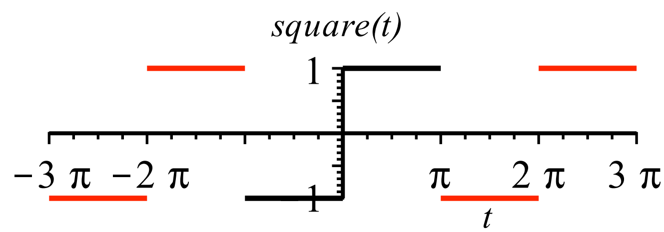
$$x''(t) + 4\,x(t) = f(t)$$
$$x(0) = 0$$
$$x'(0) = 0$$

where the forcing function is given by

$$f(t) = \begin{cases} 1 & 0 \le t < \pi \\ 0 & t \ge \pi \end{cases}.$$

9.1 Fourier series for $2\,\pi$ - periodic functions
3, 10, 30
w13.4 By computing Fourier coefficients, verify that the Fourier series for $square(t)$,
the $2\,\pi$-periodic square wave extension of

$$f(t) = \begin{cases} -1 & -\pi < t < 0 \\ 1 & 0 < t < \pi \end{cases}$$



is

$$square(t) \sim \frac{4}{\pi} \sum_{n\ odd} \frac{1}{n} \sin(n\,t).$$

9.2 General Fourier series for $2\,L$ - periodic functions
2, 9
w13.5 Notice that $square(t)$ in w13.3 is related to the $f(u)$ in 9.2.2. In fact,

$$f(u) = \frac{1}{2} + \frac{1}{2} square\left(\frac{\pi}{5} u\right).$$

a) Use this fact to re-find the answer to **9.2.2**, by using the Fourier series for $square(t)$.
b) Use technology to graph the partial sum of the first 10 non-zero terms in your Fourier sine series of $f(t)$ in 9.2.2 to verify that it is close to the graph of the square wave.