Math 2280-001
Course review

Final exam: Friday April 28, 8:00 a.m. -10:00 a.m. (I will let you begin at 7:45 a.m. if you wish.) This is the official University time and location - our lecture room LCB 215. As usual the exam is closed book and closed note, and the only sort of calculator that is allowed is a simple scientific one. You will be provided a Laplace Transform table and the formulas for calculating Fourier coefficients. The algebra and math on the exam should all be doable by hand.

The final exam will be comprehensive, but weighted to more recent material. Rough percentage ranges per chapter are below – these percentage ranges add up to more than 100% because many topics span several chapters.

Chapters:

- 1-2: 10-20% first order DEs
- 3: 15-30% linear differential equations and applications
- 4.1, 5: 30-50% linear systems of differential equations and applications
- 7: 15-25% Laplace transforms and applications
- 9.1-9.4: 10-15% Fourier series and applications to forced oscillations
- 6.1-6.4: 10-15% Non-linear autonomous systems of DE's.

On the next page is a more detailed list of the topics we've investigated this semester. They are more inter-related than you may have realized at the time, so let's discuss the connections. Then we'll work an extended problem that lets us highlight these connections and review perhaps 70% of the key ideas in this course.

1-2: first order DEs
- slope fields, Euler approximation
- phase diagrams for autonomous DEs
  - equilibrium solutions
  - stability
- existence-uniqueness thm for IVPs
- methods:
  - separable
  - linear
- applications
  - populations
  - velocity-acceleration models
  - input-output models

3 Linear differential equations
- IVP existence and uniqueness
- Linear DEs
  - Homogeneous solution space, its dimension, and why
  - superposition, $\underline{x}(t)=\underline{x}_P+\underline{x}_H$
  - linear transformations
    - aka superposition
    - fundamental theorem for solution space to L(y)=f when L is linear
  - (We use vector space concepts:
    - vector spaces and subspaces
    - linear combinations
    - linear dependence/independence
    - span
    - basis and dimension)
- Constant coefficient linear DEs
  - $\underline{x}_H$ via characteristic polynomial
    - Euler's formula, complex roots
  - $\underline{x}_P$ via undetermined coefficients
  - solving IVPs
- applications:
  - mechanical configurations
    - unforced: undamped and damped
      - cos and sin addition angle formulas and amplitude-phase form
    - forced undamped: beating, resonance
    - forced damped: $\underline{\mathbf{x}}_{sp}+\underline{\mathbf{x}}_{tr}$, practical resonance
  - Using conservation of total energy (=KE+PE) to derive equations of motion, especially for pendulum and mass-spring.

4.1, 5.1-5.7 linear systems of DEs
- first order systems of DEs and tangent vector fields.
- existence-uniqueness thm for first order IVPs
- phase portraits for systems of two linear homogeneous differential equations;
- classifications based on eigendata
- vector space theory for linear first order systems:
  - superposition, $\underline{\mathbf{x}}=\underline{\mathbf{x}}_P+\underline{\mathbf{x}}_H$
  - dimension of solution space for $\underline{\mathbf{x}}_H$.
- conversion of DE IVPs or systems to first order system IVPs.
- Constant coefficient systems and methods:
  - $\underline{\mathbf{x}}'(t)=A\underline{\mathbf{x}}$
  - $\underline{\mathbf{x}}'(t)=A\underline{\mathbf{x}}+\underline{\mathbf{f}}(t)$
  - $\underline{\mathbf{x}}''(t)=A\underline{\mathbf{x}}$ (from conservative systems)
  - $\underline{\mathbf{x}}''(t)=A\underline{\mathbf{x}}+\underline{\mathbf{f}}(t)$
  - Fundamental matrices
  - Matrix exponentials
  - Matrix exponential integrating factor for inhomogeneous systems of first order linear DEs
- applications: phase portrait interpretation of unforced oscillation problems; input-output modeling; forced and unforced mass-spring systems.

7.1-7.6: Laplace transform
- definition, for direct computation
- using table for Laplace and inverse Laplace transforms ... applications to linear differential equations and systems of differential equations from Chapters 3, 5..
- Solving linear DE (and system of DE) IVPs with
  - Laplace transform. Partial fractions, on-off, Delta function, convolutions

9.1-9.4: Fourier series
- definition, orthogonality and projection.
- Computing Fourier series from def. and rescaling known series
- Applications to forced oscillations

6.1-6.4: autonomous nonlinear systems of first order DE's.
- equilibrium solutions and stability.
- classification of equilibrium points via linearization.
- Sketching phase plane near equilibria

We can illustrate many ideas in this course, and how they are tied together by studying the following two differential equations in as many ways as we can think of.

$x''(t) + 5\,x'(t) + 4\,x(t) = 0$

$x''(t) + 5\,x'(t) + 4\,x(t) = 3\cos(2t)$