

Math 2280-001

Week 3 concepts and homework, due January 30.

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

1.5: *solving linear DE's and IVP's:*

1, 7, 8, 13, 24

**week 3.1)** In the first week's homework problem **week 1.2** you were asked to check that the functions

$$y(x) = x + C e^{-x}$$

are solutions to the differential equation

$$y'(x) = x - y + 1.$$

(Then you just checked that the given functions made both sides of the differential equation equal, but had no idea where the alleged solutions actually came from.) Now that you know how to solve linear differential equations, use the methods of section 1.5 to find these same solutions.

1.5: *modeling and solving problems with first order linear DE's.* **33,36,38** (in **36** assume that the inflow rate is 3 gallons per minute, and the outflow rate is 4 gallons per minute)

**week 3.2)** A 25-year-old woman accepts an actuarial position with a starting salary of \$70,000 per year. her salary  $S(t)$  increase exponentially at a continuous rate of 5 % per year, so that  $S(t) = 70 e^{0.05 t}$  thousand dollars per year, after  $t$  years. To save for retirement she deposits 10 % of her salary continuously into a retirement account, which accumulates interest at an annual rate of 4 % per year. Let  $A(t)$  be the amount in the retirement account after  $t$  years, with  $A(0) = 0$  thousands of dollars at the time she begins her new job.

**a)** Estimate the change  $\Delta A$  in terms of  $\Delta t$  to derive the differential equation for  $A(t)$ .

**b)** Compute the amount of money she will have in her retirement account if she retires at age 67.

1.6: *more complicated first order differential equations.* We don't actually cover this section, but there's always technology:

**week 3.3)** Consider the differential equation from 1.6.28:

$$x e^y y' = 2(e^y + x^3 e^{2x}).$$

**a)** Use the *dsolve* command in Maple to find the general solution. This is done by first loading the differential equations package, then using the *dsolve* command. For details on using Maple, read the introductory notes and commands posted on our homework page, or consult the Maple help files, or use google.

**b)** Verify (by substitution into the differential equation), that the solutions you find in **a** actually solve the differential equation. Use Maple to do this computation.

2.1:

*solving IVP's for first order autonomous DE's with quadratic right hand sides, using partial fractions:*

1, 3, 6

*deriving and solving 1st order autonomous DE's from model descriptions:*

10, 12, 23, 33 (In 23, notice this is just a logistic DE, so you may use the general solution formula.)

**week 3.4)** In problem 23, use dfield to plot the differential equation slope field for  $0 \leq t \leq 3$  and  $-50 \leq x \leq 250$ . Use the dfield option which lets you specify initial values in order to add the graph of the solution to the IVP in 23, along with the graphs of the two equilibrium solutions. Notice that as you move your mouse over the dfield plot, its location is tracked by the dfield applet. Use your mouse to find the time when the solution to the IVP in 23 satisfies  $x(t) = 100$ , and record this information. Print out a copy of your plot; add the coordinates of this intersection point at which  $x = 100$ . Is the  $t$ - coordinate of this point consistent with your work in 23b?

2.2:

*equilibria, stability, phase portraits, slope fields for autonomous first order DE's:*

5, 7, 9, 11

**week 3.5)** Consider the differential equation

$$\frac{dx}{dt} = x^4 - 9x^2.$$

**a)** Find the equilibria; draw the phase portrait;

**b)** classify the equilibria as stable, asymptotically stable, or unstable (possibly one-sided stable);

**c)** use dfield to sketch the slope field and representative solution graphs, including the graphs of the equilibrium solutions, to verify your phase portrait analysis. Include this plot in your homework.